

Homework 6

1. Show that the orbits for the Lorenz system stay inside the ellipse E of the form

$$rx^2 + \sigma y^2 + \sigma(z-2r)^2 \leq C$$

for C large enough (assuming they start inside). Hint: Choose C large that

$$\frac{x^2}{br} + \frac{y^2}{br^2} + \frac{(z-r)^2}{r^2} > 1$$

on the boundary of E .

2. (i) Show that the e -values λ for the linearization around C^+ and C^- satisfy

$$\lambda^3 + (\sigma + b + 1)\lambda^2 + (r + \sigma)b\lambda + 2b\sigma(r-1) = 0$$

(ii) Show that $\lambda = i\omega$ for ω real when $r = r_H$

$$r_H = \sigma(\sigma + b + 3) / (\sigma - b - 1)$$