Fall
2006

HW 3
Math 266A

1. Draw the phase plane for the ODE

$$u_{tt} = u(1-u)$$

This should include: stationary points and their type, energy, global flow. Show that there is a homoclinic orbit that starts and ends at $(0,0)$.

2. Consider a Hamiltonian system

$$\begin{pmatrix} q \\ p \end{pmatrix}_t = J \nabla H$$

in which $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\nabla H = \begin{pmatrix} H_q \\ H_p \end{pmatrix}$

and $H = H(p,q)$.

Show that H is a conserved quantity for this system.

At a stationary point find the linearized system in terms of $\nabla\nabla H$. In particular show a relation between the eigenvalues of $\nabla\nabla H$ and those for the flow.

3. Consider the perturbation of the ODE in (1) to

$$u_{tt} = u(1-u) + \varepsilon u_t$$

Find the stationary points and their type for all values of ε.

For ε small and negative, find the global flow features.