HW2. Math 266A

Handout: Friday Oct 13

Due : Friday Oct 20

1. Show that for any matrices A, B of small size

$$e^{\frac{1}{2}A} e^{B} e^{\frac{1}{2}A} - e^{A+B} = \cancel{O(|A|^3+|B|^3)}$$
$$= O(|A|^3 + |B|^3)$$

~~m.~~ in which $|A|$ is a norm on the matrix.

2. Let

$$A = \begin{pmatrix} -2\varepsilon & 1 \\ 0 & -\varepsilon \end{pmatrix} \qquad x_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Let $x(t)$ solve

$$x_t = Ax$$
$$x(0) = x_0$$

Find ~~the~~

$$\max_{0 \le t < \infty} |x(t)| .$$

3. Let M ~~have eigen~~ be a real matrix with eigenvalues $\pm i$,

~~Show that its orbits are ellipses~~

and consider the ODE

$$u_t = Mu$$

Show that the orbits in the phase plane are ellipses.

<u>Hint</u> Write $M = C^{-1} \Lambda C$ with $\Lambda = \mathrm{diag}(i \; -i)$. ~~Show that~~ Consider $V = CU$. Find an ODE for V and show that its orbits are circles.

4. Consider the ODE

$$\begin{pmatrix} u \\ v \end{pmatrix}_t = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

Find and classify its stationary points.