$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$y = \arctan x \;\leftrightarrow\; x = \tan y$$
$$\arctan(0) = 0$$
$$\arctan(1) = \pi/4$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{a\to x}\frac{f(a)-f(x)}{a-x}, \qquad \underbrace{y = f(a) + f'(a)(x-a)}_{\text{tangent line at } x=a}$$

$$\frac{d}{dx}\big(x^a\big) = ax^{a-1}, \quad \frac{d}{dx}\big(\sin x\big) = \cos x, \quad \frac{d}{dx}\big(\cos x\big) = -\sin x, \quad \frac{d}{dx}\big(\tan x\big) = \sec^2 x$$

$$\frac{d}{dx}\big(\sec x\big) = \sec x \tan x, \quad \frac{d}{dx}\big(\arctan x\big) = \frac{1}{1+x^2}, \quad \frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{\big(g(x)\big)^2}, \qquad \frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)\cdot g'(x)$$

Implicit derivatives
- * take derivative of both sides w.r.t. x (don't forget y' terms)
- * rearrange and solve for y'

Related rates
- * find relationship between the quantities that are changing
- * take derivative of both sides w.r.t t
- * use known values to solve

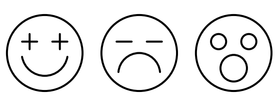
$$\underbrace{\Delta f \approx f'(a)\Delta x \;\;\text{or}\;\; f(x) \approx L(x) = f(a) + f'(a)(x-a)}_{\text{Linear approximation}} \qquad \underbrace{f'(c) = 0 \qquad f'(c) = \text{DNE}}_{\text{critical points}}$$

$$\begin{cases} f' > 0 & \leftarrow \quad f \text{ increasing} \\ f' < 0 & \leftarrow \quad f \text{ decreasing} \\ f'' > 0 & \leftarrow \quad f \text{ concave up} \\ f'' < 0 & \leftarrow \quad f \text{ concave down} \end{cases}$$

Global min/max
- * list endpoints and critical points
- * plug all values into the function
- * largest=max, smallest=min

$$\underbrace{f'(c) = \frac{f(b)-f(a)}{b-a}}_{\substack{f \text{ continuous and differentiable} \\ \text{Mean Value Theorem}}}$$

Applied optimization
- * express quantity we are optimizing as a function of a single variable
- * take derivative and find critical pts.
- * use first/second derivative test



$$\text{if } f(x) \ge g(x) \;\longrightarrow\; \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \quad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx, \quad \int_a^a f(x)\,dx = 0$$

$$\int x^a\,dx = \frac{1}{a+1}x^{a+1} + C \quad (a \neq 1), \quad \int \sin x\,dx = -\cos x + C, \quad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C, \quad \int \sec x \tan x\,dx = \sec x + C, \quad \int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \underbrace{f\big(g(x)\big)g'(x)\,dx}_{u=g(x),\; du=g'(x)\,dx} = \int f(u)\,du, \quad \frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(u)\,du\right) = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$$

$$\text{average value} = \frac{1}{b-a}\int_a^b f(x)\,dx, \qquad \text{area when } f(x) \ge g(x) = \int_a^b \big(f(x) - g(x)\big)\,dx$$

$$\text{volume of revolution}: \quad \pi\int_a^b \big((f(x))^2 - (g(x))^2\big)\,dx \quad (x\text{-axis}) \;\;\text{or}\;\; 2\pi\int_a^b x\big(f(x) - g(x)\big)\,dx \quad (y\text{-axis})$$