

MATH 3B (Butler)
Practice for Midterm II

*Try to answer the following questions without the use of book, notes or calculator.
Time yourself and try to finish the test in less than 50 minutes.*

1. Find $\int_0^e \ln x \, dx$. (Hint: L'Hospital's rule might be useful.)

2. (a) Find $\int \frac{dx}{e^{2x} - 1}$ by substituting $u = e^x$.

(b) Find $\int \frac{dx}{e^{2x} - 1}$ by substituting $u = e^{2x} - 1$.

3. The function $f(x)$ has

$$P_2(x) = \frac{1}{3} + 2x - \frac{1}{4}x^2$$

as its degree 2 Taylor polynomial around $x = 0$. Let $g(x) = \arctan(\sqrt{3}f(x))$. Find the degree 2 Taylor polynomial for $g(x)$ around $x = 0$. (Hint: $\arctan(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$, $\arctan(1) = \frac{\pi}{4}$ and $\arctan(\sqrt{3}) = \frac{\pi}{3}$.)

4. Now, the Star-Belly Sneetches had bellies with stars. The Plain-Belly Sneetches had none upon thars. Then one day Sylvester McMonkey McBean came to town with his wondrously wonderful machine, "Just one pass through, hop on board, and you will have a star for sure." The Sneetches listened and the Sneetches thought and those who wanted a star-belly stepped up and bought.

Sylvester kept track of the percentage of Sneetches with stars (Q) and noticed with time (t) in months that

$$\frac{dQ}{dt} = \frac{1}{3}(1 - Q)^2t.$$

His business was quick, he did not want to delay, and so he recalled on his very first day that $Q = \frac{1}{4}$, to make a quick buck and then leave this place he decided to leave when $Q = \frac{3}{4}$. How many months then will it take until Sylvester McMonkey McBean leaves this place?

5. Find $\int e^{\sqrt[3]{x}} \, dx$.