Equation sheet – the following may or may not be helpful.

$\sin^2 x + \cos^2 x = 1, \qquad \tan^2 x + 1 = \sec^2 x, \qquad \ln(ab) = \ln a + \ln b, \qquad \ln a^b = b \ln a$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \qquad \frac{d}{dx}\int_{g(x)}^{h(x)} f(u)\,du = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx, \qquad \int x^k\,dx = \frac{1}{k+1}x^{k+1} + C \ \ (k \neq 1), \qquad \int \frac{dx}{x} = \ln x + C$$

$$\int e^x\,dx = e^x + C, \qquad \int \sin x\,dx = -\cos x + C, \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec x \tan x\,dx = \sec x + C, \qquad \int \sec^2 x\,dx = \tan x + C, \qquad \int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\begin{matrix}\text{average}\\ \text{value}\end{matrix} = \frac{1}{b-a}\int_a^b f(x)\,dx, \qquad \begin{matrix}\text{area when}\\ f(x) \geq g(x)\end{matrix} = \int_a^b \big(f(x) - g(x)\big)\,dx$$

$$\begin{matrix}\text{volume of}\\ \text{revolution}\end{matrix} = \pi \int_a^b \big((f(x))^2 - (g(x))^2\big)\,dx, \qquad \int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du \ \ (\text{where } u = g(x))$$

$$\int u\,dv = uv - \int v\,du, \qquad \text{Partial fractions} \begin{cases} \text{* check to see if division needed} \\ \text{* factor denominator} \\ \text{* decompose into small pieces} \\ \text{* integrate each piece} \end{cases}$$

$$\begin{matrix}\text{Taylor polynomial for}\\ f(x) \text{ around } x = a\end{matrix} = P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\begin{matrix}\text{solving}\\ \text{differential}\\ \text{equations}\end{matrix} \begin{cases} \text{* separate} \\ \text{* integrate} \\ \text{* simplify} \end{cases} \qquad \frac{dy}{dx} = g(y) \text{ and } g(\widehat{y}) = 0 \text{ then } \begin{cases} g'(\widehat{y}) < 0 & \text{stable} \\ g'(\widehat{y}) > 0 & \text{unstable} \end{cases}$$

$$\begin{matrix}\mathbf{x} = [x_1, x_2, \ldots, x_n]' \\ \mathbf{y} = [y_1, y_2, \ldots, y_n]'\end{matrix} \qquad |\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}, \qquad \mathbf{x}\cdot\mathbf{y} = \sum_{i=1}^n x_i y_i = |\mathbf{x}||\mathbf{y}|\cos(\theta)$$

$$\begin{matrix}\text{Plane with point } (x_0, y_0, z_0) \\ \text{and normal vector } \mathbf{n} = [a, b, c]'\end{matrix} : \quad a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}, \quad \begin{matrix}\text{Tangent plane}\\ \text{(linear approximation)}\end{matrix} : \ z = f(x_0, y_0) + \frac{\partial f(x_0, y_0)}{\partial x}(x - x_0) + \frac{\partial f(x_0, y_0)}{\partial y}(y - y_0)$$

$$\nabla f(x, y) = \Big[\frac{\partial f(x, y)}{\partial x}, \frac{\partial f(x, y)}{\partial y}\Big]', \qquad D_{\mathbf{u}}(x_0, y_0) = \nabla f(x_0, y_0) \cdot \mathbf{u} \ \ (\mathbf{u} \text{ a unit vector})$$

$$\begin{matrix}\text{critical point}\\ \Leftrightarrow \nabla f = \mathbf{0}\end{matrix}, \qquad D = f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - \big(f_{xy}(x_0, y_0)\big)^2 \begin{cases} < 0 \text{ then saddle} \\ > 0 \text{ and } f_{xx} > 0 \text{ then min} \\ > 0 \text{ and } f_{xx} < 0 \text{ then max} \end{cases}$$

Find min/max of $f(x, y)$ given constraint $g(x, y) = c$

$\Leftrightarrow$ find critical points of $F(x, y, \lambda) = f(x, y) - \lambda\big(g(x, y) - c\big)$