Homework 7 – Due Wednesday, May 27

Section 1.3 (page 28): 3, 9

Section 1.4 (page 38): 6, 8, 12ab, 25

Chapter 1 Supplementary (page 43): 6, 30, 31, 38

Section 2.1 (page 53): 16

Supplemental problems:

1. Let G be a graph on n vertices. Show that if every vertex has degree at least $(n-1)/2$ then G is connected.

2. Jellybeans of eight different colors are in six jars. There are twenty jellybeans of each color. Prove that there must be a jar containing two pairs of jellybeans of two different colors of jellybeans. (Hint: use the pigeonhole principle.)

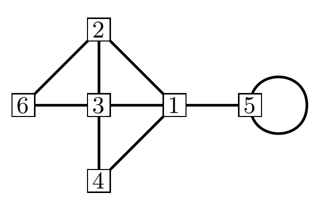
3. There is a huge bin filled with thousands of balls of many different colors. What is the smallest number n of balls you need to take from the bin so that among the n balls you must have either seven balls of one color or seven balls with no two the same color.

(Your answer needs two parts, first show that with n balls you must satisfy the condition, second show that with $n-1$ balls you might not satisfy the condition.)

4. Fullerenes are molecules composed entirely of carbon, the most famous of which is Buckminsterfullerene (C_{60}) which is composed of 60 carbon atoms in the shape of a soccer ball. These three dimensional molecules can be represented in the plane by simple connected planar graphs where each vertex corresponds to a carbon atom and edges represent bonds between atoms. In the graph corresponding to the fullerene each vertex has degree three and each face must be either a pentagon (five-sided) or a hexagon (six-sided).

Show that in every fullerene (regardless of the number of carbon atoms) there are *exactly* twelve pentagons.

5. A graph has an Eulerian cycle if there is a closed walk which uses each edge *exactly* once. A related problem is to find the shortest closed walk (i.e., using the fewest number of edges) which uses each edge *at least* once. (This is known as the "Chinese Postman" problem and comes up frequently in applications for optimal routing.) Consider the following graph on six vertices.



(a) Find a walk that starts and ends at vertex 1 of length 11 so that each edge in the graph is used at least once.

(b) Explain why 11 is the smallest number of edges needed to find a walk that starts and ends at vertex 1 and uses each edge at least once.