Homework 5 – Due Wednesday, May 6

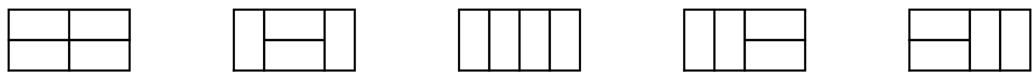
Section 7.3 (page 299): 3ad, 5, 11

Section 7.4 (page 303): 1ac, 9bc, 11, 19

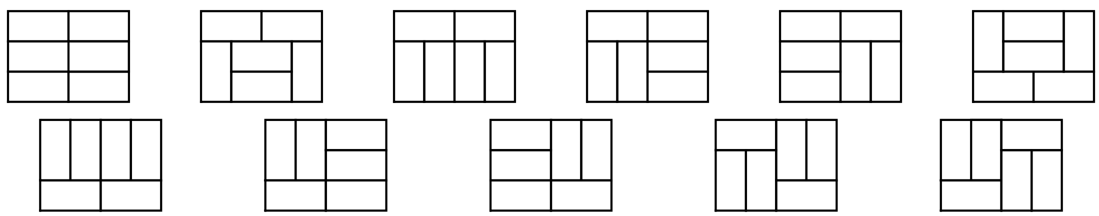
Section 7.5 (page 310): 1bd, 2bd, 3c

Supplemental problems:

1. (a) Let t_n be the number of ways to tile a $2\times n$ board using 1×2 tiles so that each square is covered and there is no overlap of tiles. Show that $t_n = F_{n+1}$, where F_n is the nth Fibonacci number. (The case $n = 4$ is shown below.)



(b) Let A_n be the number of ways to tile a $3\times 2n$ board using 1×2 tiles so that each square is covered and there is no overlap of tiles. In this problem we will solve for A_n. (The case $A_2 = 11$ is shown below.)



(i) Let B_n be the number of ways to tile a $3\times(2n-1)$ board with the 1×1 upper left corner square removed. Set up a system of recurrence relations involving A_n and B_n.

(ii) Show that the system of recurrence relations found above can be reduced to a single recursion involving A_n, namely $A_{n+2} = 4A_{n+1} - A_n$.

(iii) Verify that $A_1 = 3$. Use this and $A_2 = 11$ to solve the above recurrence.

2. Solve the recurrence relationship $d_{n+2} = \dfrac{1 + d_{n+1}}{d_n}$ for $n \geq 1$ with $d_1 = x$ and $d_2 = y$ $(x, y > 0)$.

3. For a fixed value of r consider the sequence defined by $g_n = \sum_k r^k \binom{n-k}{k}$.

(a) Show that $g_0 = 1$, $g_1 = 1$ and $g_{n+1} = g_n + rg_{n-1}$ for $n \geq 1$.

(b) Solve the recurrence for g_n in the special case $r = -1/4$.

4. Solve the recurrence relationship

$$r(n+3) = 6r(n+2) - 11r(n+1) + 6r(n) \quad \text{for } n \geq 0,$$

with initial conditions $r(0) = 5$, $r(1) = 6$ and $r(2) = 10$.

5. Solve the recurrence relationship

$$t_n = \begin{cases} t_{n-1} + 2t_{n-2} + 2 & \text{if } n \text{ is even} \\ t_{n-1} + 2t_{n-2} & \text{if } n \text{ is odd} \end{cases}$$

with initial conditions $t_0 = 0$ and $t_1 = 0$. (Hint: try to rewrite the recurrence into a single expression.)

6. Let F_n be the Fibonacci numbers (i.e., $F_0 = 0$, $F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$). Find $\displaystyle\sum_{n=0}^{\infty} \frac{F_n}{2^n}$.

(Hint: look at the generating function for the Fibonacci numbers.)