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*** UCLA Combinatorics Seminar ***

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Contraction and expansion of convex sets

Abstract

Let $\mathcal{S}$ be a set system of convex sets in $\mathbb{R}^d$. Helly's theorem states that if all sets in $\mathcal{S}$ have empty intersection, then there is a subset $\mathcal{S}' \subset \mathcal{S}$ of size $d+1$ which also has empty intersection. The conclusion fails, of course, if the sets in $\mathcal{S}$ are not convex or if $\mathcal{S}$ does not have empty intersection. Nevertheless, in this work we present Helly type theorems relevant to these cases with the aid of a new pair of operations, affine-invariant *contraction* and *expansion* of convex sets.

These operations generalize the simple scaling of centrally symmetric sets. The operations are continuous, i.e., for small $\epsilon > 0$, the contraction $C^{-\epsilon}$ and the expansion C^{ϵ} are close (in Hausdorff) to C. We obtain two results. The first extends Helly's theorem to the case of set systems with non-empty intersection:

(a) If $\mathcal{S}$ is any family of convex sets in $\mathbb{R}^d$ then there is a finite subfamily $\mathcal{S}' \subseteq \mathcal{S}$ whose cardinality depends only on ϵ and d, such that $\cap_{C \in \mathcal{S}'} C^{-\epsilon} \subseteq \cap_{C \in \mathcal{S}} C$.

The second result allows the sets in $\mathcal{S}$ a limited type of non-convexity:

(b) If $\mathcal{S}$ is a family of sets in $\mathbb{R}^d$, each of which is the union of k *fat* convex sets, then there is a finite subfamily $\mathcal{S}' \subseteq \mathcal{S}$ whose cardinality depends only on ϵ, d and k, such that $\cap_{C \in \mathcal{S}'} C^{-\epsilon} \subseteq \cap_{C \in \mathcal{S}} C$.

Joint work with Michael Langberg, Open University of Israel