AMS Special Session on Extremal and Probablistic Combinatorics
Annandale-on-Hudson, NY
October 8-9, 2005

- **József Balogh**, University of Illinois at Urbana-Champaign
  TITLE: *Non-crossing matchings and paths in randomly labeled planar point sets*
  ABSTRACT: Consider a set S of points in the plane in general position, where each point has an integer label from $\{0, 1, \ldots, n-1\}$. This naturally induces a labeling of the edges: each edge (i, j) is assigned label $i + j$, modulo n. In the spirit of harmonious graphs, we propose the algorithms for finding (hopefully) large non–crossing, pseudoharmonious matchings or paths, i. e. the matchings or paths in which no two edges have the same label. When the point labels are chosen uniformly at random, and independently of each other, our matching algorithm with high probability (w.h.p.) delivers a nearly–perfect matching, a matching of size $n/2 - O(n^{1/3} \ln n)$. We show that, in sharp contrast, a near-perfect path is unlikely: w.h.p. the length of the longest path is below $0.96n$. This is joint work with Boris Pittel and Gelasio Salazar

- **Tom Bohman**, Carnegie Mellon University
  TITLE: *A note on the Karp-Sipser algorithm*
  ABSTRACT: Let G be a graph on n vertices chosen uniformly at random for the collection of all graphs that have some fixed degree distribution. We consider the Karp-Sipser algorithm applied to G, which produces a matching. (In each step of the Karp-Sipser algorithm we choose an edge incident with a vertex of degree 1 to be an edge in the matching if such an edge exists and otherwise choose a random edge.) In this talk we give a simple condition on the degree distribution that ensures that the algorithm produces a nearly perfect matching. Applications of this observation to the problem of finding Hamilton cycles in various models of sparse random graphs will also be discussed. This is joint work with Alan Frieze.

- **Alan Frieze**, Carnegie Mellon University
  TITLE: *The game chromatic number of random graphs*
  ABSTRACT: Given k colours and a graph G, two players Maker and Breaker play alternately. Maker tries to properly colour the graph and Breaker tries to produce a proper partial colouring that cannot be extended to a full colouring. The game chromatic number of G is the minimum k for which Maker has a winning strategy. The parameter was introduced by Bodlaender. As the title indicates, we study the case where G is a random graph or a random bipartite graph. This is joint work with Tom Bohman and Benny Sudakov.

- **Penny Haxell**, University of Waterloo
  TITLE: *An algorithmic version of the hypergraph regularity method*
  ABSTRACT: Extending the celebrated Szemerédi Regularity Lemma for graphs, in 2002 Frankl

and Rödl proved a Regularity Lemma for 3-uniform hypergraphs that has a corresponding Counting Lemma, allowing counting of small substructures. The joint application of these two lemmas is called the Hypergraph Regularity Method and has led to many results on hypergraph problems.

We give an algorithmic version of the Regularity Lemma for 3-uniform hypergraphs, together with a corresponding Counting Lemma, and discuss some applications. This is joint work with B. Nagle and V. Rödl.

- **Jeff Kahn**, Rutgers University
  TITLE: *Some correlation inequalities*
  ABSTRACT: We show positive association (i.e. positive correlation of increasing events) for some families of random variables arising in percolation, the random cluster model and the "fractional fuzzy Potts model".

- **Peter Keevash**, Caltech
  TITLE: *Pairwise intersections and forbidden configurations*
  ABSTRACT: Let $f_m(a,b,c,d)$ denote the maximum size family of a family $\mathcal{F}$ of subsets of an m-element set so that there is no pair $A, B \in \mathcal{F}$ with

$$|A \cap B| \geq a, \quad |\bar{A} \cap B| \geq b, \quad |A \cap \bar{B}| \geq c, \quad |\bar{A} \cap \bar{B}| \geq d.$$

  By symmetry we can assume $a \geq d$ and $b \geq c$. We show that $f_m(a,b,c,d)$ is $\Theta(m^{a+b-1})$ if either $b > c$ or $a, b \geq 1$. We also show $f_m(0,b,b,0)$ is $\Theta(m^b)$ and $f_m(a,0,0,d)$ is $\Theta(m^a)$. This can be viewed as a result concerning forbidden configurations, and provides further evidence for a conjecture of Anstee and Sali.

  Our key tool is a strong stability version of the Ahlswede-Khachatrian Complete Intersection Theorem, which is of independent interest. This is joint work with Richard Anstee.

- **Jeong Han Kim**, Microsoft Research
  TITLE: *Giant component and Poisson Cloning model*
  ABSTRACT: Since P. Erdős and A. Rényi introduced random graphs in 1959-1968, the theory of random graphs has played a central role in probabilistic combinatorics. The emergence of the giant component is one of a few subjects with the most colorful history in the area. After introducing a new (equivalent) model for the random graph, called the Poisson cloning model, we improve and/or reprove various results regarding the giant component. For instance, the following theorem, which improves Łuczak's result, can be proven:

  Theorem. Let $p = (1+\varepsilon)/n$ with $n^{-1/3} \ll \varepsilon < 1$ and $1 \ll \alpha \ll (\varepsilon^3 n)^{1/2}$. Then, with probability $1 - e^{-\Omega(\alpha^2)}$, the random graph $G(n,p)$ has the largest component of size between

$$\theta_\varepsilon n - \alpha(n/\theta_\varepsilon)^{1/2} \quad \text{and} \quad \theta_\varepsilon n + \alpha(n/\theta_\varepsilon)^{1/2},$$

  where θ_ε is the larger solution for the equation $1 - \theta - e^{-\theta(1+\varepsilon)} = 0$.

If time arrows, we will discuss a more general problem, namely, the problem of the giant component of the graph that is the union of a fixed graph and a random graph.

- **Dhruv Mubayi**, University of Illinois at Chicago
  TITLE: *Co-degree density of the Fano plane*
  ABSTRACT: I will introduce co-degree problems for hypergraphs, and present a proof of the following result: Each 3-uniform hypergraph on n vertices, in which every pair of vertices lies in at least $(1/2 + o(1))n$ triples, contains a copy of the Fano plane.

- **Oleg Pikhurko**, Carnegie Mellon University
  TITLE: *On Product Anti-Magic Graphs*
  ABSTRACT: Let us call a graph G with m edges *product anti-magic* if we can bijectively label edges with numbers $1, \dots, m$ so that no two vertices have the same product over incident edges. Ringel conjectured that every connected graph of order at least 4 is product anti-magic. We prove this conjecture for all graphs of sufficiently large order by using probabilistic methods.

- **Radoš Radoičić**, Rutgers University
  TITLE: *On the diameter of separated point sets with many nearly equal distances*
  ABSTRACT: A point set is *separated* if the minimum distance between its elements is one. We call two real numbers *nearly equal* if they differ by at most one. We prove that for any dimension $d \geq 2$ and any $\gamma > 0$, if P is a separated set of n points in $\mathbb{R}^d$ such that at least γn^2 pairs in $\binom{P}{2}$ determine nearly equal distances, then the diameter of P is at least $C(d, \gamma)n^{2/(d-1)}$ for some constant $C(d, \gamma) > 0$. In the case of $d = 3$, this result confirms a conjecture of Erdős. The order of magnitude of the above bound cannot be improved for any d. Our proof includes regularity lemma and Ramsey-type arguments. This is a joint work with J. Pach and J. Vondrák.

- **Jacques Verstraete**, University of Waterloo
  TITLE: *Product Representations of Polynomials*
  ABSTRACT: In this talk I will discuss the algorithmic problem of efficiently determining the existence of a linear dependence amongst a set of vectors in a finite dimensional vector space over F_q. To do so, a more general framework is introduced, where we look for integer factorizations of points in the value set of a polynomials.

  For a polynomial $f \in Z[X]$ and positive integers k and N, let $\rho_k(N; f)$ denote the maximum size of a set $A \subset \{1, 2, \dots, N\}$ such that no product of k distinct elements of A is in the value set of f.

  Using a little algebraic geometry, the probabilistic method and some extremal combinatorics, we prove that for every polynomial f of prime degree d, either $\rho_k(N; f)$ is linear in N, or $|f|$ is the d^{th} power of a monic linear polynomial and $\rho_k(N; f) \sim c\pi(N) + O(N^{1-1/2d})$ and c is completely determined. This generalizes earlier results of Erdős (1963), Erdős, Sós and A. Sárközy (1995), Györi and G. Sárközy (1997). We conclude with some open questions.

- **Jan Vondrák**, Microsoft Research

  TITLE: *Ramsey-type results for the hypercube*

  ABSTRACT: We consider the question of existence of monochromatic cycles for edge colorings of the hypercube, raised by Fan Chung in 1992. She proved that for any fixed $k \geq 2$ and an even $\ell \geq 4$, for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic cycle of length 2ℓ. On the other hand, there is a 2-coloring of any hypercube which avoids monochromatic cycles of length 4, and a 3-coloring which avoids monochromatic cycles of length 6. Fan Chung asked what happens for odd $\ell \geq 5$, i.e. whether it is possible to avoid monochromatic cycles of length $10, 14, 18, \ldots$.

  We answer this question by proving that for any fixed $k \geq 2$ and $\ell \geq 5$, any k-coloring of a sufficiently large hypercube contains a monochromatic cycle of length 2ℓ. More generally, we provide a characterization of all subgraphs of the hypercube with this Ramsey property. In addition, we show the existence of subgraphs H_k such that for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic copy of H_k but this is not the case for all $(k+1)$-colorings. This is joint work with Noga Alon, Radoš Radoičić and Benny Sudakov.

- **Yi Zhao**, Georgia State University

  TITLE: *On the VC-dimension of uniform hypergraphs*

  ABSTRACT: In the early 70's, Sauer,Perles-Shelah, Vapnik-Chervonenkis independently proved that if a set system $\mathcal{F}$ on $[n]$ contains more than $\binom{n}{0} + \cdots + \binom{n}{d-1}$ sets, then there exists a *shattered* d-element set S of $[n]$, namely, $\{E \cap S : E \in \mathcal{F}\}$ contains all subsets of S. Using the algebraic method, Frankl and Pach showed that for r-uniform $\mathcal{F}$ ($r \geq d$), the same holds whenever $\mathcal{F} > \binom{n}{d-1}$ and conjectured that $\mathcal{F} > \binom{n-1}{d-1}$ suffices. But this was later disproved by Ahlswede and Khachatrian. In this talk, we show that if $d = 2^t + 1$ for positive integer t, then every d-uniform $\mathcal{F}$ on $[n]$ of size $\binom{n}{d-1} - \lg n$ forces a shattered d-element set. We also note that there are infinite many construtions achieving the same effect as the one of Ahlswede and Khachatrian. This is a joint work with Dhruv Mubayi.