HW#7: due Mon, 11/27/2017
The numbering refers to online version of the textbook

Problem 1: Exercise 3.1.3, page 83

Problem 2: Exercise 3.2.2, page 85

Problem 3: Exercise 3.2.3, page 86

Problem 4: Consider i.i.d. random variables $X_1, \ldots, X_{2n+1}$ with continuous probability density $f$ and let $\hat{X}_1 < \cdots < \hat{X}_{2n+1}$ denote their a.s. unique reordering. Let $m$ denote the median of the law of $X_1$, defined, e.g., as $\sup\{x \in \mathbb{R}: P(X_1 \leq x) \leq 1/2\}$. Prove that

$$\sqrt{n}(\hat{X}_{n+1} - m) \xrightarrow{\text{law}} \mathcal{N}(0, \sigma^2)$$

where $\mathcal{N}(\mu, \sigma^2)$ is the normal random variable with mean zero and variance $\sigma^2$. Identify $\sigma^2$ in terms of $f$. Prove all claims.

Problem 5: Exercise 3.2.6, page 90

Problem 6: Exercise 3.2.7, page 90

Problem 7: Exercise 3.2.12, page 90

Problem 8: Exercise 3.2.13, page 90

Problem 9: Exercise 3.2.15, page 90

Problem 10: Exercise 3.2.16, page 90

Problem 11: Let $\{\mu_n: n \in \mathbb{N}\}$ and $\mu$ be probability measures on $\mathbb{R}$ and let $F_n(x) := \mu_n((-\infty, x])$ and $F(x) := \mu((-\infty, x])$ be their associated CDFs. Prove that

$$\mu_n \xrightarrow{w} \mu \iff F_n(x) \xrightarrow{n \to \infty} F(x) \quad \forall x \in \{ \text{continuity points of } F \}$$

Hint: Use freely the Portmanteau Theorem and or parts of the proof thereof.

Problem 12: Exercise 3.3.8, page 96

Problem 13: Exercise 3.3.17, page 100