Problem 9: As it turns out, there is a sequence \( \{a_n\}_{n \in \mathbb{N}} \) enumerating all rationals, i.e., \( \mathbb{Q} = \{a_n: n \in \mathbb{N}\} \). Define \( f: \mathbb{R} \to \mathbb{R} \) by
\[
f(a_n) := \frac{1}{n+1}, \quad n \in \mathbb{N},
\]
and
\[
f(x) := 0, \quad x \in \mathbb{R} \setminus \mathbb{Q}.
\]
Prove that \( f \) is discontinuous at every \( x_0 \in \mathbb{Q} \) and continuous at every \( x_0 \in \mathbb{R} \setminus \mathbb{Q} \).