Problem 1: Let $f: X \to Y$ be a function and $\{Y_\alpha: \alpha \in I\}$ a collection of subsets of $Y$. Prove the following equalities:

$$f^{-1}\left(\bigcup_{\alpha \in I} Y_\alpha\right) = \bigcup_{\alpha \in I} f^{-1}(Y_\alpha)$$

and

$$f^{-1}\left(\bigcap_{\alpha \in I} Y_\alpha\right) = \bigcap_{\alpha \in I} f^{-1}(Y_\alpha)$$

Problem 2: Find a close-form formula for $\sum_{k=1}^{n} k^4$. Prove your formula by induction.

Problem 3: Ex 1.6, Page 5

Problem 4: Ex 1.8, Page 5

Problem 5: Ex 1.10, Page 6

Problem 6: Let $Q^+ := \{p/q: p, q \in \mathbb{N}, q > 0\}$. Find a suitable choice of zero element $0 \in Q^+$ and successor function $S: Q^+ \to Q^+$ such that the Peano axioms hold except that of induction.

Bonus question: Can this choice be made so that all five Peano axioms hold? (We will answer this later in class.)

Problem 7: Ex 2.2, Page 12

Problem 8: Ex 2.8, Page 13