A MODEL FOR PARTICLE LADEN FLOW IN A SPIRAL CONCENTRATOR

Sungyon Lee*[a)], Yvonne Stokes**& Andrea L. Bertozzi*
*Dept. of Mathematics and Applied Mathematics Laboratory, University of California, Los Angeles, CA 90095
**School of Mathematical Sciences, The University of Adelaide, South Australia, 5005, Australia

Summary We apply recent results for gravity driven slurries to the model of a spiral concentrator in the case of a monodisperse particle slurry. We use a thin film approximation to derive an equilibrium profile for the particle concentration and fluid thickness. Our results explain observations seen in laboratory experiments and commercial products.

INTRODUCTION

Spiral concentrators are used in the mining industry to separate particles of different size or density. The existing modeling literature considers the flow as a background fluid carrying non-neutrally buoyant particles. However recent work on modeling of slurries on inclines shows that at relatively modest volume fractions of particles, the presence of the particles affects the flow and, moreover, interparticle interactions such as hindered settling and shear-induced migration can quantitatively explain the dynamics of the separation of particle mixtures under gravity. We incorporate this physics into a model for particle segregation in a spiral concentrator.

We consider a mixture of heavy particles and a viscous liquid flowing down a helical channel of radius A and pitch $2\pi P$, with a rectangular cross section, as shown Figure 1(a). The particle and liquid mass densities obey $\rho_p > \rho_\ell$ and the liquid bulk viscosity is μ_ℓ. The volume fraction of particles is denoted by ϕ. Individual particles as well as the liquid phase are incompressible. All lengths have been scaled by the half width of the channel, so that the channel has width 2. The slope of the channel is $\tan\alpha = P/A = \tau/\epsilon$, where $\epsilon = A/(A^2+P^2)$ and $\tau = P/(A^2+P^2)$ are the helix curvature and torsion, respectively. For a spiral concentrator, $\epsilon \sim 0.5$ and $\tau \sim 0.1$, so that we assume $\tau \ll 1$ and, for the coordinate system adopted, $\epsilon < 1$. Furthermore the flow is, typically, of small depth relative to the channel width, characterised by the dimensionless parameter $\delta \ll 1$.

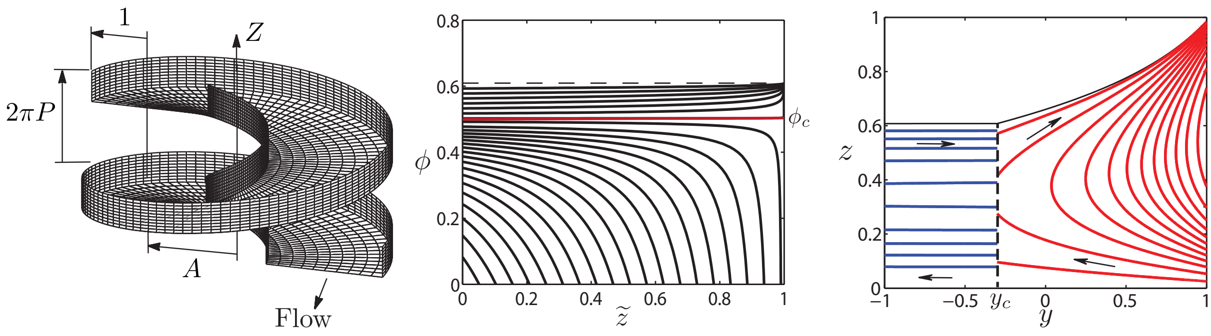

Figure 1. (a) A helical channel of rectangular cross section. (b) Particle concentrations in the scaled $\tilde{z}$ direction ($\tilde{z} = z/h$) for various total concentrations, ϕ_0, at the inclination angle, $\alpha = 25^\circ$. There is a unique ϕ_c, denoted by the horizontal line, such that the particles are uniformly distributed in $\tilde{z}$ and the net flux in y is zero. (c) Streamlines in a helical channel of rectangular cross section for parameters $\alpha = 25^\circ$, $\epsilon = 0.5$, $\tau = 0.1$, $\delta = 0.1$, $Q = 0.2$, $Q_p = 0.0015$. The domain is separated into two regions at $y = y_c$: the particle-heavy region $-1 \le y < y_c$ ($\phi(y,z) = \phi_c$), and the clear fluid region $y_c < y \le 1$ ($\phi = 0$). We note that the transverse flow shown here is secondary to the dominant downstream flow; however, it plays an important role in particle separation.

EQUILIBRIUM THIN-FILM MODEL

We seek a steady-state solution that is independent of distance along the channel, reducing the problem to a two-dimensional domain. We simplify the Navier-Stokes and particle transport equations by a perturbation expansion in terms of the torsion τ and δ. Let x be the direction of the primary flow down the channel and y and z be the axes in the channel cross section, with z being directed upwards. In this (x, y, z) coordinate system the (dimensionless) velocity is (u, v, w) and p is pressure. To leading order in τ and δ the model is

$$\frac{\partial v}{\partial y} + \frac{\epsilon v}{1+\epsilon y} + \frac{\partial w}{\partial z} = 0, \tag{1a}$$

$$\frac{\partial \sigma}{\partial z} + \rho \sin\alpha = 0, \quad \sigma = \mu(\phi)\frac{\partial u}{\partial z}, \tag{1b}$$

[a)] Corresponding author. E-mail: sungyon@math.ucla.edu

$$-\frac{\partial p}{\partial z} - \rho\cos\alpha = 0, \quad -\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) + \frac{\rho\chi}{1+\epsilon y}u^2 = 0, \quad \chi = \frac{\delta K}{2R}, \tag{1c}$$

$$\frac{-2\cos\alpha}{9K_{coll}}\rho_s(1-\phi) - \phi\frac{\partial\sigma}{\partial z} = \left[1 + 2\left(\frac{K_{visc}-K_{coll}}{K_{coll}}\right)\left(\frac{\phi}{\phi_{max}-\phi}\right)\right]\sigma\frac{\partial\phi}{\partial z} = 0, \tag{1d}$$

where we have set $\delta R/F^2 = 1$, and $R = \rho U a/\mu_\ell$ and $F = U/\sqrt{ga}$ are the Reynolds and Froude numbers, respectively; $K = 2\epsilon R^2$ is the Dean number. The viscosity μ and density ρ of the fluid-particle mixture depend on the particle volume fraction ϕ as follows:

$$\mu(\phi) = \left(1 - \frac{\phi}{\phi_{max}}\right)^{-2}, \qquad \rho = 1 + \rho_s\phi, \quad \rho_s = \frac{\rho_p - \rho_\ell}{\rho_\ell}.$$

The constant ϕ_{max} is the maximum possible volume fraction of particles. Boundary conditions are $u = v = w = 0$ on the channel wall $z = 0$ and, on the free surface $z = h(y)$: $\partial u/\partial z = 0$, $\partial v/\partial z = 0$, $p = 0$, and $w = v(dh/dy)$. In the particle-transport equation (1d) we have fluxes due to gravitational sedimentation and shear-induced migration as in [1, 2, 3]. In particular, shear-induced migration includes the effects of particle collisions and gradient in the effective suspension viscosity, $\mu(\phi)$, and it plays an essential role in describing key properties of the suspension flow [1]. Neglect of Brownian motion is justified by a large Péclet number. For further details on the derivation of the flow model see [6, 7] and for the particle transport model see [1, 2].

RESULTS

Equations (1b) and (1d) are solved simultaneously at each horizontal position y for a given inclination angle α and total volume fraction of particles $\phi_0(y) = \int_0^h \phi(z,y)dz$. For each value of y, (1d) may be written in terms of a new variable $\widetilde{z} = z/h(y)$, such that a solution may be found over $0 \leq \widetilde{z} \leq 1$ that is independent of $h(y)$. The particle transport problem in this form is exactly the inclined plane problem of [3, 5], and Figure 1(b) shows examples of $\phi(\widetilde{z})$ for different values of $\phi_0(y)$. *Note that there is a specific value of $\phi_0(y)$ such that the particle concentration is constant (i.e. $\phi = \phi_c$) in the $\widetilde{z}$ direction.* The steady state solution for the particle laden flow requires zero net flux in the y direction, both for fluid and for total fluid-particle mixture: $\int_0^{h(y)} v(y,z)dz = 0$, $\int_0^{h(y)} v(y,z)\phi(y,z)dz = 0$, respectively. Given $v(y,z)$ satisfying the first of these conditions, the only possible $\phi(y,z)$ from the full set of curves (Figure 1(b)) to satisfy the second is the constant case, $\phi = \phi_c$. If ϕ is monotone decreasing in z (lower portion of Figure 1(b)), then the net flux in y is negative (towards the center of the spiral). Likewise if ϕ is monotone increasing in z (upper portion of Figure 1b), the net flux is positive (away from the center of the spiral). Based on this simple argument, we assume a form of steady state solution where the cross-sectional domain is divided into two distinct regions at a critical position $y = y_c$: region A, $-1 \leq y < y_c$, where the particle concentration is constant everywhere, ($\phi(y,z) = \phi_c$), and region B, $y_c < y \leq 1$ of clear fluid ($\phi = 0$). For given total flux Q (fluid and particles), flux of particles Q_p and channel angle α, and requiring the free surface height at y_c to be continuous, we are able to solve for the position y_c, free surface $h(y)$, and the flow; the resultant streamlines are shown in Figure 1(c). The clear separation of particle-laden and clear fluid, in the case of a monodisperse slurry flow, has been observed in our own preliminary laboratory experiments and in commercial applications [8].

CONCLUSIONS

In summary we propose an equilibrium solution to the particle flow problem in a helical channel, in which the particles all collect near the center of the spiral, consistent with observations of heavy particles in spiral concentrator geometry. In the future, we will model particles of different densities to study the separation of a polydisperse slurry. This will allow us to identify the parameter regime that optimizes the separation in the helical channel, which will be important in the mining and various industry applications.

References

[1] Murisic N., Ho J., Hu V., Latterman P., Koch T., Lin K., Mata M., and Bertozzi A.L., Particle-laden viscous thin-film flows on an incline: experiments compared with an equilibrium theory based on shear-induced migration and particle settling, *Physica D*, **240**(20):1661-1673, 2011.

[2] Cook B.P., Bertozzi A.L., and Hosoi A.E., Shock Solutions for Particle-Laden Thin Films, *SIAM J. Appl. Math.*, **68**(3):760-783, 2008.

[3] Cook B.P., Theory for particle settling and shear-induced migration in thin-film liquid flow, *Phys. Rev. E*, **78**:045303, 2008.

[4] Holland-Batt A.B., A method for the prediction of the primary flow on large diameter spiral troughs, *Miner. Eng.*, **22**:352-356, 2009.

[5] Murisic N., Pausader B., Peschka D., and Bertozzi A.L., Dynamics of particle settling and resuspension in viscous liquids, preprint, 2012.

[6] Stokes Y.M., Wilson S.R., Duffy B.R.: Thin-film flow in open helically-wound channels. In M. Behnia, W. Lin, G.D. McBain (eds), *15th Australian Fluid Mechanics Conference (CSROM)*, 2004, Paper AFMC00187.

[7] Stokes Y.M., Duffy B.R., Wilson S.R., and Tronnolone H., Flow down a rectangular helical channel, preprint, 2012.

[8] http://www.alicoco.com/index.php