

# GRAPH MBO METHOD FOR MULTICLASS SEGMENTATION OF HYPERSPECTRAL STAND-OFF DETECTION VIDEO

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## ABSTRACT

We consider the challenge of detection of chemical plumes in hyperspectral image data. Segmentation of gas is difficult due to the diffusive nature of the cloud. The use of hyperspectral imagery provides non-visual data for this problem, allowing for the utilization of a richer array of sensing information. In this paper, we present a method to track and classify objects in hyperspectral videos. The method involves the application of a new algorithm recently developed for high dimensional data. It is made efficient by the application of spectral methods and the Nyström extension to calculate the eigenvalues/eigenvectors of the graph Laplacian. Results are shown on plume detection in LWIR standoff detection.

*Index Terms*— classification, tracking, MBO scheme, Nyström extension method, hyperspectral data

## 1. INTRODUCTION

Detecting chemical plumes in the atmosphere is a problem that can be applied to many areas, such as defense, security and environmental protection. If the airborne toxins are identified accurately, one can combat the use of chemical gases as weapons, prevent fatalities due to accidental leakage of toxic gases and avoid contamination of the atmosphere. Identification of harmful gases with high fidelity is needed to provide warnings in threatening situations. In these grave scenarios, it is crucial to accurately track the diffusion of dangerous plumes into the atmosphere. Laboratory measured signatures of toxic chemicals are available to assist in chemical plume identification. However, testing and training data is not readily available due to the inherent danger of these real world situations. Instead, open air testing with surrogate chemicals is conducted to study the diffusion of chemical plumes. The developed plume detection methods must meet strict requirements to ensure the fidelity of a detector.

We propose applying the method outlined in [1] to hyperspectral data, in particular, to track and classify chemical plumes, recorded in a hyperspectral video sequence. The pixels of the images in the video are considered as vertices in

a graph, and we minimize the total variation with fidelity to known data. The Nyström extension method is used to efficiently calculate eigenfunctions of the graph Laplacian. They are then used both for operator assisted assignment of fidelity values and in the actual total variation minimization algorithm itself. The paper is organized as follows: section 2 reviews the graph representation of the data as well as the Nyström extension method, section 3 presents the method and the results, and the section 4 contains the conclusions.

We consider the data set, described in more detail in [2], composed of video sequences recording the release of chemical plumes at the Dugway Proving Ground. The data was provided by the Applied Physics Laboratory at Johns Hopkins University. The images are of dimension  $128 \times 320 \times 129$ , where the last dimension indicates the number of channels, each depicting a particular frequency from 7,820 nm to 11,700 nm, spaced 30 nm apart. The sets of images were taken from videos captured by three long wave infrared (LWIR) spectrometers, each placed at a different location about 2 km away from the release of plume at an elevation of around 1300 feet. One hyperspectral image is captured every five seconds. This data set has been studied in other works such as [3], [4], [5]. Prior work on hyper spectral plume detection using other sensors includes [6] (MWIR) and [7] (HYDICE). This paper is the first example of the new graphical MBO scheme applied to standoff detection data. The results are excellent compared to prior work in this area.

There are many challenges to be faced when tracking chemical plumes. One obstacle faced by the authors of [3] is the significant preprocessing needed to accurately detect the plume. Due to the noisy structure of the data set, principal component analysis reduced the data to three main features used to produce a false color video sequence of the plume release, followed by midway equalization to smooth the flicker between frames. Similar preprocessing is used in [4], which outlines a novel plume detection method involving a binary partition tree. The advantage of our method is that it does not require any preprocessing of the hyperspectral data; we use the raw data organized in a graph setting. Moreover, as pointed out in [4], the ground truth data is nonexistent, since surrogate chemicals, instead of the toxic ones, are used in testing. This makes the assessment of the results somewhat difficult. The authors of [3] dealt with this problem by using

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an adaptive matched subspace detector (AMSD) described in [7] to benchmark their spectral clustering results. AMSD is a probabilistic detection scheme that uses a generalized likelihood ratio test to choose between two hypotheses: target present or absent.

## 2. GRAPHICAL REPRESENTATION OF THE DATA AND EIGENVECTOR COMPUTATION

Graph-based methods often use the discrete Laplace operator to exploit underlying similarities in the data set. For example, the approach of spectral clustering involves the calculation of the eigenvectors of the graph Laplacian, which are then used to segment the data.

We represent the data as nodes in a weighted graph, with each edge assigned a measure of similarity  $w(i, j)$  between each pair of vertices it is connecting. Let  $\mathbf{W}$  be the matrix  $W_{ij} = w(i, j)$ , and the degree of a vertex  $i \in V$  be defined as  $d_i = \sum_{j \in V} w(i, j)$ . If  $\mathbf{D}$  is the diagonal matrix with elements  $d_i$ , then the graph Laplacian is expressed as  $\mathbf{L} = \mathbf{D} - \mathbf{W}$ .

Papers, such as [8], [1] and [9] observe that a scaled graph Laplacian is computationally a better choice in the case of large same size. To make use of efficient numerical linear algebraic methods, we implement the symmetric operator

$$\mathbf{L}_s = \mathbf{D}^{-\frac{1}{2}} \mathbf{L} \mathbf{D}^{-\frac{1}{2}} = \mathbf{I} - \mathbf{D}^{-\frac{1}{2}} \mathbf{W} \mathbf{D}^{-\frac{1}{2}}. \quad (1)$$

We wish to form a graph that utilizes information inherent in the data over time, so a method of utilizing data from multiple time steps was implemented. This method of performing multiframe analysis is done by selecting  $k$  different video frames and then concatenating the data points, allowing for data to be associated over these  $k$  frames. The computation of the graph Laplacian for all of these pixels results in a very large matrix. As an example, the Dugway Proving Ground hyperspectral data is of size  $128 \times 320$  pixels with 129 spectral bands. For seven frames, the full graph Laplacian is a square matrix of length 36,986,880. Thus, a method for quickly computing eigenfunctions of the graph Laplacian is desired. Utilizing the Nyström method, we are able to quickly compute an accurate approximation to the eigenfunctions. The Nyström method was formed to approximate the eigenfunctions of the graph Laplacian without the hassle of computing the full Laplacian. This is done by obtaining a small sample set from the data and performing matrix completion that utilizes properties of eigenfunctions to complete the Laplacian. The algorithm described in [10] and [5] is:

1) Randomly select  $k$  data points to form the set  $A$ , while the rest of data forms the set  $B$  consisting of  $n$  data points. The best utilization of Nyström has  $n \gg k$ , so the sample set is much smaller than the rest of the data.

2) Compute distances amongst data in  $A$ , named  $D_A$  and distances between data in  $A$  and  $B$ , named  $D_B$ .

3) Approximate the distances amongst data in  $B$ ,  $D_C$ , with  $D_C = D'_B D_A^{-1} D_B$ . Thus, the approximate graph Laplacian,  $L$ , is given by  $L = \begin{bmatrix} D_A & D_B \\ D'_B & D_C \end{bmatrix}$ , where  $D'_B$  denotes  $D_B$  transpose.

4) Compute the row sum of the matrix  $L$ , where  $d_i = \sum_{j=1}^n L_{i,j}$ .

5) Normalize the elements of  $D_A$  and  $D_B$ , named  $\overline{D_A}$  and  $\overline{D_B}$ , respectively.

6) Setting  $Q = \overline{D_A} + \overline{D_A}^{-.5} * \overline{D_B} * \overline{D_B}' * \overline{D_A}^{-.5}$ , find the singular value decomposition,  $Q = USV'$ .

7) Compute  $V = \begin{bmatrix} \overline{D_A} \\ \overline{D_B}' \end{bmatrix} \overline{D_A}^{-.5} U L^{-.5}$

8) Compute the eigenvector approximation  $Eig = \frac{V_i}{\sqrt{1 - L_{ii}^{-.5}}}$

Figure 1 shows a sampling of four different eigenvectors. Note that each eigenvector highlights a different aspect of the image; for example, the third eigenvector outlines the plume. In addition, the background is maintained through the seven different frames. The total run-time for the Nyström extension with 100 eigenvectors is less than one minute on a 2.8 GHz Intel Core 2 Duo. Below we use these eigenvectors for two parts of our multi-class clustering algorithm.

## 3. MBO CLUSTERING OF HYPERSPECTRAL DATA

In [1], an algorithm is developed to efficiently solve the multiclass assignment problem for graph-based data sets. It is semi-supervised and thus needs ‘‘ground truth’’ assignments for part of the data set. Since we lack real ground truth, we perform operator assisted spectral clustering to obtain partial ground truth. This is performed by identifying the relevant eigenfunctions and thresholding at some level to identify a subset of pixels that are highly likely to be part of the chosen class. For this segmentation we choose four classes: plume, sky, foreground, and mountain. We denote the class label vector as the matrix  $\mathbf{u}$ . The node  $i$  adopts a composition of states  $\mathbf{u}_i \in \mathbb{R}^K$ . Here  $K = 4$  for four class segmentation. The  $k^{th}$  component of  $\mathbf{u}_i$  is the probability the node belongs to class  $k$ . For each node  $i$ , we require the vector  $\mathbf{u}_i$  to be an element of the Gibbs simplex  $\Sigma^K$ , defined as

$$\Sigma^K := \left\{ (x_1, \dots, x_K) \in [0, 1]^K \mid \sum_{k=1}^K x_k = 1 \right\}. \quad (2)$$

Vertex  $k$  of the simplex is given by the unit vector  $\mathbf{e}_k$ .

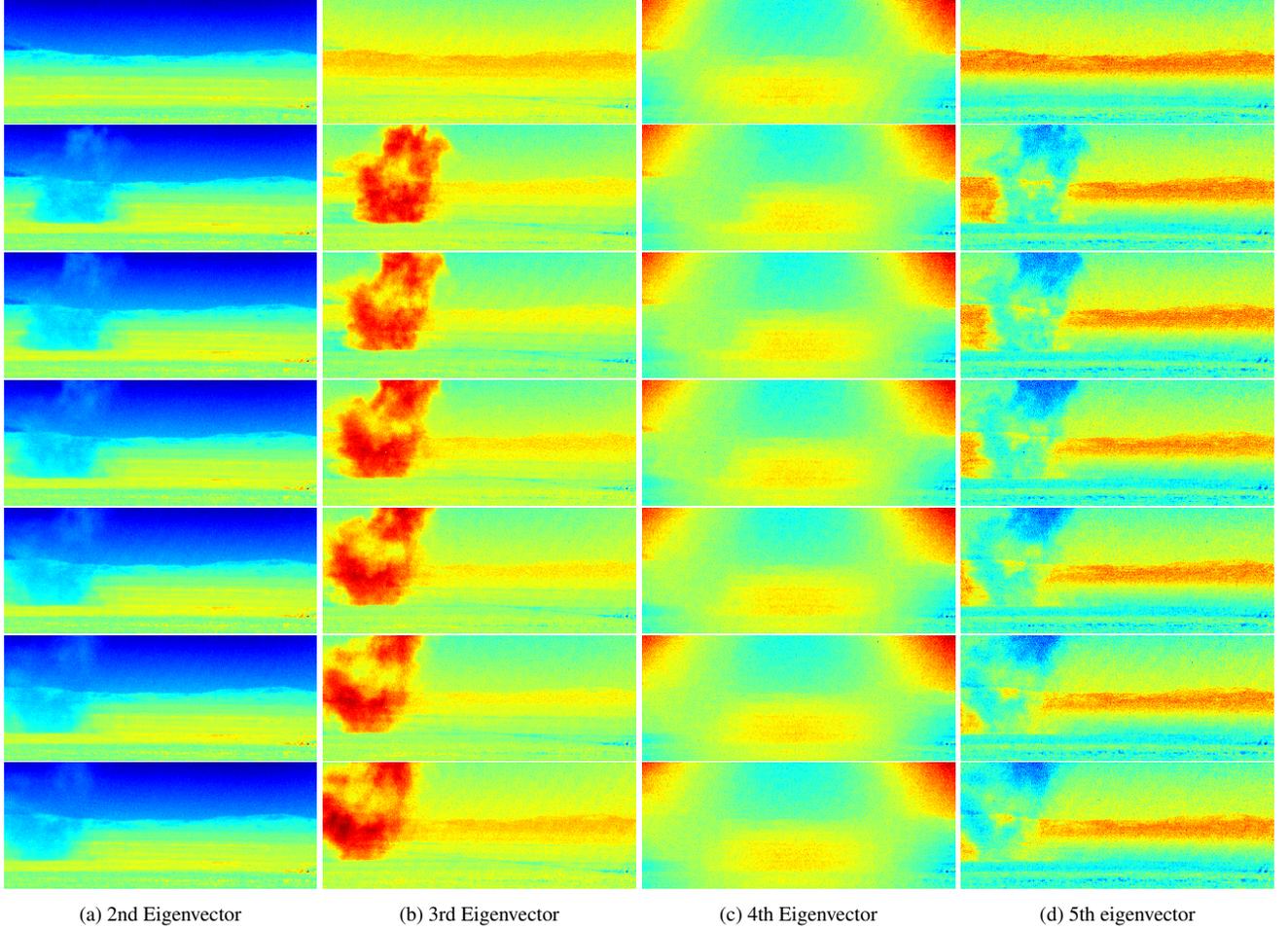
It is shown in [1] that alternating between the following two steps results in an efficient classification algorithm:

1. Heat equation with forcing term:

$$\frac{\mathbf{u}^{n+\frac{1}{2}} - \mathbf{u}^n}{dt} = -\mathbf{L}_s \mathbf{u}^{n+\frac{1}{2}} - \boldsymbol{\mu}(\mathbf{u}^n - \hat{\mathbf{u}}). \quad (3)$$

2. Thresholding:

$$\mathbf{u}_i^{n+1} = \mathbf{e}_k, \quad (4)$$



**Fig. 1:** Eigenvectors of the symmetric normalized graph Laplacian (1) for the 7 video frames shown in false color.

where  $e_k$  is the vertex in the simplex closest to the projection of  $\mathbf{u}_i^{n+\frac{1}{2}}$  onto the simplex using [11].

Here  $\mu_i$  is a positive constant  $\mu$  if node  $i$ 's label is known beforehand (fidelity point) and 0 otherwise, and  $\hat{\mathbf{u}}_i$  is a vector indicating prior class knowledge of sample  $i$ . Also, note that in the second step the row vector  $\mathbf{u}_i^{n+\frac{1}{2}}$  of step 1 is projected back to the simplex before any thresholding takes place. This is done because the result of step 1 is not necessary an element of the Gibbs simplex. As discussed in [8] and [1], the method is the MBO scheme for classification motivated by the following variational problem:

$$\min_{\mathbf{u} \in \Sigma^k} |\mathbf{u}|_{TV} + \frac{\mu}{2} \|\mathbf{u} - \hat{\mathbf{u}}\|^2, \quad (5)$$

a least squares fit to the known ‘‘ground truth’’ data and total variation term that minimizes the length of the graph cut.

Scheme (3) is solved using the eigenvalue/eigenvector decomposition of the symmetric graph Laplacian. The Laplacian term is treated implicitly. The first part of the algorithm

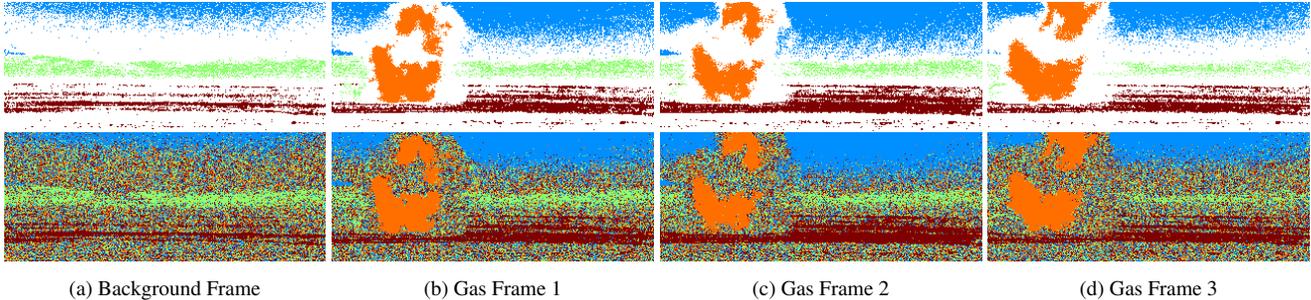
can be rewritten as

$$\mathbf{u}^{n+\frac{1}{2}} = (\mathbf{I} + dt \mathbf{L}_s)^{-1} (\mathbf{u}^n - dt \mu (\mathbf{u}^n - \hat{\mathbf{u}})). \quad (6)$$

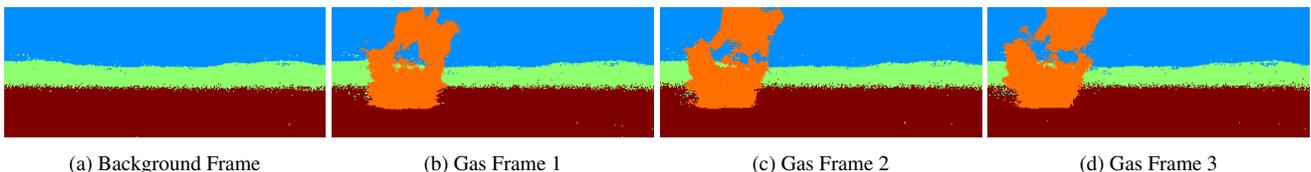
We use the eigendecomposition  $\mathbf{L}_s = \mathbf{X} \mathbf{\Lambda} \mathbf{X}^T$  to write

$$\mathbf{I} + dt \mathbf{L}_s = \mathbf{X} (\mathbf{I} + dt \mathbf{\Lambda}) \mathbf{X}^T, \quad (7)$$

but we approximate  $\mathbf{X}$  by a truncated matrix retaining only  $N_e$  eigenvectors ( $N_e \ll N_D$ ), to form a matrix of dimension  $N_D \times N_e$ . The term in the parenthesis in (7) is a diagonal  $N_e \times N_e$  matrix. This allows one to calculate  $\mathbf{u}^{n+\frac{1}{2}}$  rapidly. In particular, we write, for the  $n^{\text{th}}$  iteration,  $\mathbf{u}^n = \mathbf{X} \mathbf{a}^n$  and  $\mu (\mathbf{u}^n - \hat{\mathbf{u}}) = \mathbf{X} \mathbf{d}^n$ , where  $\mathbf{a}$  and  $\mathbf{d}$  are matrices of dimension  $N_e$  by  $K$ , where  $K$  is the number of classes. Denote  $\mathbf{E}$  to be the diagonal matrix containing the eigenvalues of the symmetric graph Laplacian, then  $\mathbf{L}_s \mathbf{u}^n = \mathbf{X} \mathbf{E} \mathbf{a}^n$ . Also denote by  $\mathbf{a}_k$  and  $\mathbf{d}_k$  the  $k^{\text{th}}$  row of  $\mathbf{a}$  and  $\mathbf{d}$ , respectively. Plugging all the known expressions into (3), we obtain an equation for  $\mathbf{a}_k^{n+1}$  that effectively replaces (3):



**Fig. 2:** First 4 frames of the hyper spectral video: (top row) operator assisted ‘ground truth’ results from spectral clustering - used as fidelity points in the MBO clustering algorithm; (bottom row) initialization for the MBO scheme. Classification is denoted by color: green = mountain; blue = sky; brown = foreground; orange = plume. White pixels denote unclassified pixels.



**Fig. 3:** First 4 frames, Results of the MBO classification algorithm. Color as specified in figure 2.

$$\mathbf{a}_k^{n+1} = \frac{\mathbf{a}_k^n - dt\mathbf{d}_k^n}{1 + dt\lambda_k} \quad (8)$$

where  $\lambda_k$  is the  $k^{th}$  eigenvalue of the symmetric graph Laplacian. The remaining step is simple thresholding.

We tested our method on seven video frames, using four classes, to segment the plume, sky, foreground and mountain. The fidelity region is shown in the first row of figure 2. The initialization for the MBO scheme is displayed in the second row. The final segmentation results, after 17 iterations, are shown in figure 3, using first four frames.

The fidelity region is calculated differently from [1], where the fidelity points were chosen randomly from known ground truth data. Without the ground truth, we use an operator assisted method involving spectral clustering. In particular, by thresholding appropriately the values of eigenvectors, one obtains information about a particular class. For example, as shown in Figure 1, the third eigenvector provides information about the plume. By thresholding its values, one can find the pixels that are most likely part of the plume. We used the fifth eigenvector to obtain fidelity for the mountain, and the second one for both the sky and the foreground. This process resulted in 36% points of the overall points from all the frames identified as as good fidelity points.

For the MBO scheme, we start with an initialization of randomly chosen phase classes for non-fidelity points and the ‘‘ground truth’’ value for the fidelity points. The MBO iteration was performed 17 times using a stopping criterion for convergence, with  $dt = 0.1$  and  $\mu = 100$ . We compared

results with 10 to 100 eigenvectors; they deteriorated with less than 10 eigenvectors but were similar with more than 10 eigenvectors. In all tests we used the same operator assisted fidelity points. The MBO iteration took around 11 seconds (for 100 eigenvectors) on a 2.4 GHz Intel Core i2 Quad, after obtaining the eigenvectors from the Nyström scheme.

#### 4. CONCLUSION

We presented an application of a recent multiclass classification algorithm [1] to hyperspectral video data. We use the Nyström extension method to efficiently calculate the needed eigenvectors. This implementation of the algorithm requires an operator assisted spectral clustering preprocessing step to identify a subset of pixels denoted as ‘‘ground truth’’ for the four classes. The resulting classification of chemical plumes and background pixels are excellent. Only a small number of eigenvectors, ten in particular, is needed to achieve a good result and no preprocessing is necessary. The entire process took about a minute on desktop PCs.

## 5. REFERENCES

- [1] C. Garcia-Cardona, E. Merkurjev, A.L. Bertozzi, A. Flenner, and A. Percus, "Fast multiclass segmentation using diffuse interface methods on graphs," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2014.
- [2] J.B. Broadwater, D. Limsui, and A.K. Carr, "A primer for chemical plume detection using LWIR sensors," Tech. Rep., National Security Technology Department, 2011.
- [3] T. Gerhart, J. Sunu, L. Lieu, E. Merkurjev, J.-M. Chang, J. Gilles, and A. L. Bertozzi, "Detection and tracking of gas plumes in LWIR hyperspectral video sequence data," in *SPIE Conference on Defense Security and Sensing*, 2013.
- [4] G. Tochon, J. Chanussot, J. Gilles, M. Dalla Mura, J.-M. Chang, and A.L. Bertozzi, "Gas plume detection and tracking in hyperspectral video sequences using binary partition trees," preprint, 2014.
- [5] J. Sunu, J.-M. Chang, and A.L. Bertozzi, "Simultaneous spectral analysis of multiple video sequence data for LWIR gas plumes," in *SPIE Conference on Defense, Security, and Sensing*, 2014, accepted.
- [6] M. Hinnrichs, J. O. Jensen, and G. McAnally, "Hand-held hyperspectral imager for standoff detection of chemical and biological aerosols," in *Optical Technologies for Industrial, Environmental, and Biological Sensing*. International Society for Optics and Photonics, 2004, pp. 67–78.
- [7] D. Manolakis, C. Siracusa, and G. Shaw, "Adaptive matched subspace detectors for hyperspectral imaging applications," in *Acoustics, Speech, and Signal Processing, 2001. Proceedings.(ICASSP'01). 2001 IEEE International Conference on*. IEEE, 2001, vol. 5, pp. 3153–3156.
- [8] E. Merkurjev, T. Kostic, and A. L. Bertozzi, "An MBO scheme on graphs for segmentation and image processing," *SIAM Journal of Imaging Sciences*, vol. 6, no. 4, pp. 1903–1930, 2013.
- [9] A.L. Bertozzi and A. Flenner, "Diffuse interface models on graphs for classification of high dimensional data," *Multiscale Modeling & Simulation*, vol. 10, no. 3, pp. 1090–1118, 2012.
- [10] C. Fowlkes, S. Belongie, F. Chung, and J. Malik, "Spectral Grouping Using the Nyström Method," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 26, no. 2, 2004.
- [11] Y. Chen and X. Ye, "Projection onto a simplex," *arXiv preprint arXiv:1101.6081*, 2011.