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PREFACE TO SPECIAL ISSUE ON MATHEMATICS OF SOCIAL SYSTEMS

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This special issue is an outgrowth of a minisyposium titled "Mathematics of Social Systems" held at the 9th AIMS Conference on Dynamical Systems, Differential Equations and Applications, held in Orlando, FL in July 2012. Presenters from that session were invited to submit papers that were reviewed using the usual procedures of the DCDS journals, along with additional authors from the field. Mathematics has already had a significant impact on basic research involving fundamental problems in physical sciences, biological sciences, computer science and engineering. Examples include understanding of the equations of incompressible fluid dynamics, shock wave theory and compressible gas dynamics, ocean modeling, algorithms for image processing and compressive sensing, and biological problems such as models for invasive species, spread of disease, and more recently systems biology for modeling of complex organisms and complex patterns of disease. This impact has yet to come to fruition in a comprehensive way for complex social behavior. While computational models such as agent-based systems and well-known statistical methods are widely used in the social sciences, applied mathematics has not to date had a core impact in the social sciences at the level that it achieves in the physical and life sciences. However in recent years we have seen a growth of work in this direction and ensuing new mathematics problems that must be tackled to understand such problems. Technical approaches include ideas from statistical physics, nonlinear partial differential equations of all types, statistics and inverse problems, and stochastic processes and social network models. The collection of papers presented in this issue provides a backdrop of the current state of the art results in this developing new research area in applied mathematics. The body of work encompasses many of the challenges in understanding these discrete complex systems and their related continuum approximations.

The paper [3] by Bedrossian and Rodríguez considers a general class of nonlocal partial differential equations with nonlinear diffusion and nonlocal advection. A special class of such problems, namely the parabolic-elliptic Patlak-Keller-Segel (PKS) model for chemotaxis is well known in the literature. However, more general models arise in problems involving social aggregation dynamics with first order attraction and nonlinear diffusion, typical of anti-crowding dynamics. In a previous paper [4] the authors developed a unifying theory for the Patlak-Keller-Segel global existence theory with the local existence and uniqueness theory for less singular nonlocal models in dimensions $d \geq 3$. However many biologically relevant examples

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are precisely the case $d = 2$ which is treated here. The authors also consider the case of spatial inhomogeneities in the rate at which chemo-attractant diffuses and decays. Such inhomogeneities are present in every physical system and thus it is important to understand to what degree they affect the overall qualitative behavior of the PKS model.

The paper [12] considers an agent-based model of urban crime that takes into account repeat or near repeat victimization. The continuum limit of this agent-based model is the two-component reaction-diffusion PDE system

$$A_t = \delta\Delta A - A + PA + \alpha \quad \tau P_t = D\nabla\cdot(\nabla P - \frac{2P}{A}\nabla A) - PA + \nabla - \alpha, \tag{1}$$

where the positive constants δ, D, α, γ, and τ are all spatially independent. In this model P represents the density of criminals, A represents the attractiveness of the environment to burglary or other criminal activity, and the chomtactic drift term $-2D\nabla\cdot(P\frac{\nabla A}{A})$ represents the tendency of criminals to move towards sites with higher attractiveness. In addition, α is the baseline attractiveness, whereas $(\gamma - \alpha)/\tau$ is the constant rate of reintroduction of criminals after an event. This model was first introduced in [16] and further analyzed in [17, 18] using weakly nonlocal bifurcation theory. In this new work, singular perturbation techniques are used to construct steady-state hot-spot patterns in one and two-dimensional spatial domains, and new types of nonlocal eigenvalue problems are derived that determine the stability of these hot-spot patterns to $O(1)$ time-scale instabilities. The paper [24] considers the same model in the case of localized police deterrence. A model adds a new parameter - namely the deterrence function that depends on the total number of police - a fixed quantity - that is assumed to be optimally spatially distributed to suppress crime. When only modest police resources are available, the optimal police allocation is in the vicinity of the hotspots, leaving other parts of the city without protection. Steady state solutions are computed for various resources levels along with their stability with respect to perturbation.

The burglary model above assumes very simple decision rules regarding occurrence of criminal activity based on environmental attractiveness functions that change due to recent events. More complex decisions rules are sometimes required in models for criminal activity that involves nontrivial social interaction, such as organized crime and human dynamics in close environments. Two papers in this issue that analyze such models are [15] and [5]. In [19] Short *et al* introduce a game theoretic model for organized crime involving four classes of individuals - Informants, Villians, Apathetics, and Paladins. Crimes occur according to a Poisson arrival process and criminals are chosen for the crime at random from a pool of $I + V$ informants and villains. Then, for each crime, a victim is selected at random from the remaining players. There is a detailed play process that includes option to report crimes and the option to change one's player type. There is a course grain dynamics that is well-approximated by a coupled system of four ordinary differential equations for the population size of the different population types. The system will progress to a Utopian state whenever the number of informants is greater than zero. If there are no informants then the system can evolve to either Utopia or Dystopia. The work published here in this volume [15] by McCalla considers this same model but with a spatial dependence. He accomplishes this by adding peer pressure - in which spatial neighbors can effect an agent's behavior through a diffusive effect. This revises the model to the form of a coupled system of nonlinear diffusion equations. The author finds interesting traveling wave solutions akin to invasion waves in invasive species models [23, 13]. Both analysis and simulations are presented. In [5] Burger *et al* present an optimal control approach to modeling fast exit scenarios in pedestrian crowds. Each person moves according to a rule in which they minimize a cost functional that depends on position, velocity, exit time, and the overall density of people. The microscopic interaction rules leads to a mean-field limit that is a parabolic optimal control problem. They related the optimal conditions to the Hughes model for pedestrian flow and derive both analytical and numerical results related to this class of models.

A well-studied topic in the last decade is that of swarming and there are many interesting agent based models that have been studied including the Viscek model [22], the Cucker-Smale model [7] and the second order model studied in [8]. These models all exhibit complex collective pattern formation and one typically needs a continuum theory to develop analytical predictive results about these social systems. Mackey *et al* [14] consider a first order kinematic model that can be written as a gradient flow of an energy composed of all pairwise interactions between particles coupled by radially symmetric potentials. This problem is well-studied in the literature and recently Kolokolnikov *et al* derived stability and pattern formation analysis for ring-patterns in two dimensions [11]. The Mackey paper develops new theory for this problem in the case of two distinct classes of particles. The bidisperse problem yields much more complex pattern formation than the single species problem, although similar, yet more complicated analysis of the stability problem, is possible. Both analytical results and computations are presented. Balagué *et al* [2] likewise consider the kinematic problem by studying the macroscopic continuum equations for single species particles. They are interested in the confinement properties of solutions - when solutions remain compactly supported in a ball depending on the initial data and the potential. This problem is related in spirit to the H-stability analysis first presented for the related discrete dynamic model in [8]. Comparisons are made between numerical simulations of particle dynamics and the rigorous theory. Barbaro and Degond [1] develop a hydrodynamic model from the kinetic description of a particle system that combines a noisy Cucker-Smale consensus force with self-propulsion. In the limit of large self-propulsion, they provide evidence of a phase transition from disordered to order motion that manifests itself as a change of type of the limiting equations (from hyperbolic to diffusive) at a critical noise intensity.

Another body of work studied here involve discrete time events on social networks. These include the papers [9, 20], and [6]. The paper [9] by Ghosh and Lerman considers social network analysis from the point of view of dynamic processes. By analogy with well-known PDE models, the authors classify processes on networks as either conservative or non-conservative. They relate these two classes to well-known measures of centrality used in network analysis: PageRank and Alpha-Centrality. This paper demonstrates, by ranking users in online social networks used for broadcasting information, that non-conservative Alpha-Centrality leads to a better agreement with an empirical ranking scheme than the conservative PageRank measure. These results suggest that we must broaden our class of measurement tools for social networks beyond the random walk model where a node interacts with one other node at a time. The paper [20] by Short, Mohler, Brantingham, and Tita, introduces a point process model for inter-gang violence. Such behavior is typically triggered by retaliation and is a core feature of gang activity. The authors consider an interesting inhibitory mechanism, that of multi-party inhibition. They use a coupled system of state-dependent jump stochastic differential equations to model the conditional intensities of the directed network of gang rivalries, and then analyze a continuum formulation of this model to determine stable network topologies. They fit the model to gang violence data provided by the Los Angeles Police Department, to measure the levels of excitation and inhibition present in gang violence dynamics and to estimate the stability of gang rivalries in Los Angeles. The paper [6] by Cho, Galstyan, Brantingham, and Tita addresses the fact that social network data is generally incomplete with missing information about nodes and their interactions. They propose a spatial-temporal latent point process model that describes geographically distributed interactions between pairs of entities. They assume the interactions are not fully observable and that certain interaction events lack information about participants. They develop an efficient approximate algorithm based on variational expectation maximization to infer unknown participants in an event given the location and time of the event. They validate the model with real world data. This is a new area of research in data analysis with recent related work using variational [21] and statistical approaches [10].

REFERENCES

[1] A. B. T. Barbaro and P. Degond, Phase transition and diffusion among socially interacting self-propelled agents, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1249–1278.

[2] D. Balagué, J. A. Carrillo and Y. Yao, Confinement for repulsive-attractive kernels, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1227–1248.

[3] J. Bedrossian and N. Rodríguez, Inhomogeneous Patlak-Keller-Segel models and aggregation equations with nonlinear diffusion in R^d, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1279–1309.

[4] J. Bedrossian, N. Rodríguez and A. L. Bertozzi, Local and global well-posedness for aggregation equations and Keller-Segel models with degenerate diffusion, *Nonlinearity*, **24** (2011), 1683–1714.

[5] M. Burger, M. Di Francesco, P. A. Markowich and M. T. Wolfram, Mean field games with nonlinear mobilities in pedestrian dynamics, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1311–1333.

[6] Y. S. Cho, A. Galstyan, P. J. Brantingham and G. Tita, Latent self-exciting point process model for spatial-temporal networks, *Discrete and Continuous Dynamical Systems*, **34** (2014),1335–1354.

[7] F. Cucker and S. Smale, Emergent behavior in flocks, *IEEE Transactions on Automatic Control*, **52** (2007), 852–862.

[8] M. R. D'Orsogna, Y. L. Chuang, A. L. Bertozzi and L. Chayes, Self-propelled particles with soft-core interactions: patterns, stability and collapse, *Phys. Rev. Lett.*, **96** (2006), 104302.

[9] R. Ghosh and K. Lerman, Rethinking Centrality: The role of dynamical processes in social network analysis, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1355–1372.

[10] R. A. Hegemann, E. A. Lewis and A. L. Bertozzi, An estimate & Score Algorithm for simultaneous parameter estimation and reconstruction of incomplete data on social networks, *Security Informatics*, **2**, 1–13.

[11] T. Kolokolnikov, H. Sun, D. Uminsky and A. L. Bertozzi, Stability of ring patterns arising from two-dimensional particle interactions, *Phys. Rev. E*, **84** (2011), 015203.

[12] T. Kolokolnikov, M. J. Ward and J. Wei, The stability of steady-state hot-spot patterns for a reaction-diffusion model of urban crime, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1373–1410.

[13] M. A. Lewis, B. Li and H. F. Weinberger, Spreading speed and linear determinacy for two-species competition models, *J. Math. Biol.*, **45** (2002), 219–233.

[14] A. Mackey, T. Kolokolnikov and A. L. Bertozzi, Two-species particle aggregation and stability of co-dimension one solutions, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1411–1436.

[15] S. G. McCalla, Paladins as predators: Invasive waves in a spatial evolutionary adversarial game, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1437–1457.

[16] M. B. Short, M. R. D'Orsogna, V. B. Pasour, G. E. Tita, P. J. Brantingham, A. L. Bertozzi and L. B. Chayes, A statistical model of criminal behavior, *Math. Models. Meth. Appl. Sci.*, **18** (2008), 1249–1267.

[17] M. B. Short, A. L. Bertozzi and P. J. Brantingham, Nonlinear patterns in urban crime - hotpsots, bifurcations, and suppression, *SIAM J. Appl. Dyn. Sys.*, **9** (2010), 462–483.

[18] M. B. Short, P. J. Brantingham, A. L. Bertozzi and G. E. Tita, Dissipation and displacement of hotpsots in reaction-diffusion models of crime, *Proc. Nat. Acad. Sci.*, **107** (2010), 3961–3965.

[19] M. B. Short, P. J. Brantingham and M. R. D'Orsogna, Cooperation and punishment in an adversarial game: How defectors pave the way to a peaceful society, *Phys. Rev. E*, **82** (2010), 066114.

[20] M. B. Short, G. O. Mohler, P. J. Brantingham and G. E. Tita, Gang rivalry dynamics via coupled point process networks, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1459–1477.

[21] A. Stomakhin, M. B. Short and A. L. Bertozzi, Reconstruction of missing data in social networks based on temporal patterns of interactions, *Inverse Problems*, **27** (2011), p. 115013.

[22] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen and O. Shochet, Novel type of phase in a system of self-driven particles, *Phys. Rev. Lett.*, **75** (1995), 1226–1229.

[23] X.-S. Wang, H. Wang and J. Wu. Traveling waves of diffusive predator-prey systems: Disease outbreak propagation, *Discrete Contin. Dyn. Syst.*, **32** (2012), 3303–3324.

[24] J. R. Zipkin, M. B. Short and A. L. Bertozzi, Cops on the dots in a mathematical model of urban crime and police response, *Discrete and Continuous Dynamical Systems*, **34** (2014), 1479–1506.

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