Agent-based models for crowd survival: Stability analysis and the effects of contagion and boundaries

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Jesús Rosado   Theodore Kolokolnikov

August 8, 2012

Outline

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Crowd Survival

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[1] 

[1] http://www.nationalgeographicstock.com/comp/B10/376/988955.jpg

Crowd Survival

1  2 

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Crowd Survival

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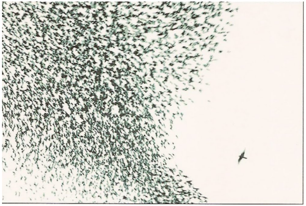
3 

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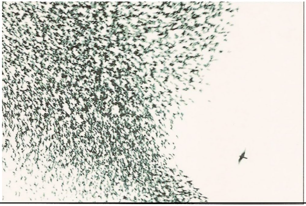
3 

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[4] http://securitymarketingguru.com/wp-content/uploads/2009/09/Sharks-and-Sardine-Bait-Ball-2-300x199.jpg

What is a contagion

Examples of contagions

- Fear
- Heat
- Emotion

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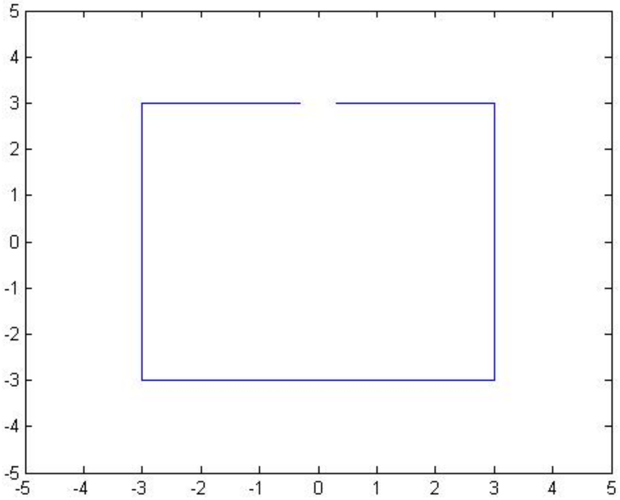

The Model

Assumption

There is a goal or set of goals that people want to reach

Assumption

There are regions of discomfort defined by a function g(x), s.t. if g(x)>g(x') then a person would rather move through x' than x

Assumption

The speed of any given person is dependent on the density of where they expect to be. People in low density move fast, people in high density move slowely

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Cost Field

- The cost field has a base cost of β
- The cost field includes the discomfort field

$$C(x) = \beta + \gamma g(x)$$

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Finding the shortest path

The solution to the Eikonal Equation can be used to find the shortest path given this cost field.

$$\|\nabla u\| = C$$

- $u = 0$ in the goal set
- Shortest path follows the gradient of u

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Basic Model

$$x_i = x_i + dt * \frac{\rho_{max} - \rho_i}{\rho_{max}} * v_i$$
$$v_i' = v_i + dt * (\nabla\phi(x_i) - v_i)$$

where ρ_i is caluclated as follows

$$xtemp_i = x_i + 5 * dt * v_i$$
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Collision Avoidance

If $\|x_i - x_j\| < 0.25$ then apply a repulsion factor of $\frac{1}{\|x_i - x_j\|}$. However if $\|x_i - x_j\| < dt$ then we only apply a repulstion factor of $\frac{1}{\sqrt{dt}}$

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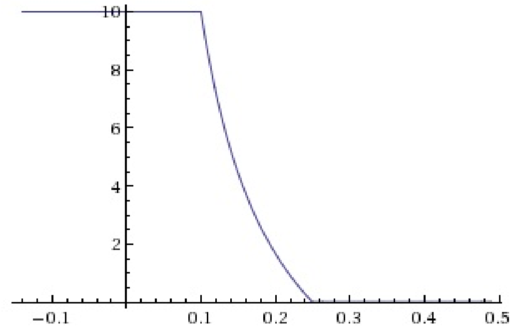

Showing Eikonal Equation solves shortest path

$$\min_P \int_P C(x)ds$$

Let us parameterize $\quad P = f(t), t \in [0,1]$

$$\min_f \int_0^1 C(f(t))\|\dot{f}(t)\|dt = \int_0^1 I(f, \dot{f})$$

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Euler-Lagrange Equation

$$\text{E-L} = \frac{\partial I}{\partial f} - \frac{d}{dt}\left(\frac{\partial I}{\partial \dot{f}}\right)$$

$$\nabla C(f)\|\dot{f}\| - \frac{d}{dt}\left(C(f)\frac{\dot{f}}{\|\dot{f}\|}\right)$$

$$\nabla C(f)\|\dot{f}\| - \left[(\nabla C(f)\cdot\dot{f})\frac{\dot{f}}{\|\dot{f}\|} + C(f)\frac{\|\dot{f}\|\ddot{f} - \left(\frac{\dot{f}}{\|\dot{f}\|}\cdot\ddot{f}\right)\dot{f}}{\|\dot{f}\|^2}\right]$$

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$$\nabla C(f)\|\dot{f}\| \cdot \dot{f}^{\perp} - \left[(\nabla C(f) \cdot \dot{f}) \frac{\dot{f}}{\|\dot{f}\|} \dot{f}^{\perp} + C(f) \frac{\|\dot{f}\|\ddot{f} - \left(\frac{\dot{f}}{\|\dot{f}\|} \cdot \ddot{f}\right)\dot{f}}{\|\dot{f}\|^2} \dot{f}^{\perp} \right]$$

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$$(\dot{f}^{\perp} \cdot \nabla C(f)) \|\dot{f}\|^2 = (\nabla u^{\perp} \cdot \nabla C) C^2$$

$$= (\{-u_y, u_x\} \cdot \nabla \|\nabla u\|) (C^2)$$

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$$(\ddot{f} \cdot \dot{f}^{\perp}) C = (\dot{\nabla} u \cdot \nabla u^{\perp}) C$$

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What have we shown

- $(\dot{f}^{\perp} \cdot \nabla C(f))\|\dot{f}\|^2 = (\ddot{f} \cdot \dot{f}^{\perp})C$
- $\implies$ E-L $= 0$
- When $\dot{f} = \nabla u$
- $\implies$ f is a minimizer of $\int_0^1 I(f)dt$

What have we shown

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Exiting The Room

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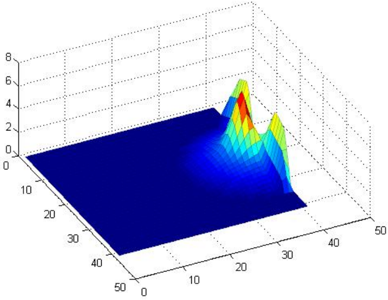
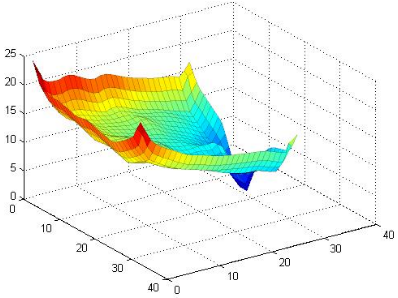
How to exit the room





Pred = [0,2]		Pred = [0,0]		Pred = [0,-2]	
Toggle	Regular	Toggle	Regular	Toggle	Regular
80.4667	80.9000	81.1667	78.6333	87.6333	81.7000

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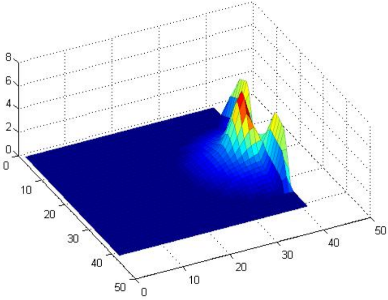

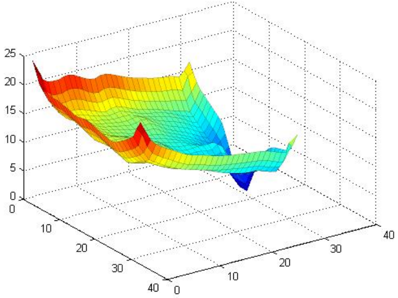

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Predator in the room

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Outline

Intro

Our goal was to develop a predator prey model with contagion while achieving certain objectives.

Objectives:

- Predator to go after nearest prey
- Prey to run away from nearest Predator
- Speed of Prey depends on level of fear
- Prey exhibit flocking behavior
- Prey gain fear when close to Predator
- Fear can spread among the Prey

Model-Predator

$$\frac{dx_i}{dt} = v_{x_i}$$
$$\frac{dv_{x_i}}{dt} = -v_{x_i} + A\frac{y_j - xi}{|y_j - x_i|} \; s.t. \; |y_j - x_i| = min\{|y_s - x_i| : s = 1..n\}$$

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Model-Prey

$$\frac{dy_i}{dt} = (1 + 2q_{y_i})v_{y_i}$$

$$\frac{dv_{y_i}}{dt} = -v_{y_i} + \chi_{\{r\leq 1\}}(|y_i - x_j|)\frac{y_i - x_j}{|y_i - x_j|^{1+2q_{y_i}}} + \chi_{\{r>1\}}(|y_i - x_j|)\frac{y_i - x_j}{|y_i - x_j|^2} + \frac{1}{n}\sum_{j\neq i} F(|y_j - y_i|)\frac{y_i - y_j}{|y_i - y_j|} s.t.\ |y_i - x_j| = min\{|y_i - x_s| : s = 1..m\}$$

$$\frac{dq_{y_i}}{dt} = \frac{B}{n}\sum_{j\neq i}\frac{q_{y_j} - q_{y_i}}{\epsilon + |y_i - y_j|^2} + \frac{1}{D}\sum_{|x_j - y_i|\leq C}\frac{\frac{1}{2} - q_{y_j}}{|x_j - y_i|} + \frac{1}{E}\sum_{|x_j - y_i|>C}(-q_{y_i})$$

$$F(r) = \frac{1}{r} - r$$

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Video of 2D Model(1 Predator)

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Why add Fear?

(Loading Video...)

(Loading Video...)

Video of 3D Model(1 Predator)

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Video of 3D Model(2 Predators)

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Outline

N preys v.s. 1 predator

- N preys, eg. sardines

$$\frac{dv_j}{dt} = -v_j + \frac{1}{N}\sum_{k=0,k\neq j}^{N-1} F(\|x_j - x_k\|)\frac{x_j - x_k}{\|x_j - x_k\|} + H(\|x_j - z\|)(x - z), \quad (1)$$

$$\frac{dx_j}{dt} = v_j \qquad j = 1...N, \quad (2)$$

where x_j's and v_j denote the position and velocity of each prey particle j, respectively.

- Predator, eg. shark

$$\frac{du}{dt} = -u + \frac{c}{N}\sum_{j=1}^{N} I(\|x_j - z\|)(x_j - z), \quad (3)$$

$$\frac{dz}{dt} = u \qquad j = 1...N, \quad (4)$$

where z and u denote the position and velocity of the predator, respectively, and c is a parameter.

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Videos

When $F(r) = r^a - r^2$, for $|a| < 1$, at the steady state, the preys form an equally-spaced ring.

1. Take $a = 0.5$, $H(r) = \frac{1}{r^2}$, $c = 50$, $I(r) = r^{0.05}\chi_{\{\bar{r}>2\}} + \frac{1}{r^5+1}\chi_{\{\bar{r}<2\}}$
2. Take $F(r) = r^{0.5} - r^2$, $H(r) = \frac{1}{r^2}$, $c = 50$, $I(r) = \frac{1}{r^5+1}$

1 2

Videos

- When $F(r) = \frac{1}{r} - r$, $H(r) = \frac{1}{r^2}$, $c = 50$, $I(r) = \frac{1}{r^2+1}$

Background

- In previous papers, the stability of ring pattern at steady state has been studied[5] .
- The evolution of uniform density in certain domains with Newtonian potential has been analyzed[6], we use the similar method to analyze our problems;
- In our model, the difference is that we add interaction with predator into the system.

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[5] T.K., H.S., D.U., and A. B.. Stability of ring patterns arising from two-dimensional particle interactions. Phys. Rev. E, 84:015203, Jul 2011

[6] T.K., Y.H., and M.P.. Singular patterns for an aggregation model with a confining potential. to appear

Steady State

We consider the prey particles locate at an equally spaced ring and the predator locates at the center of the ring at the steady state. For $j = 1...N$, taking $x_j = Re^{\frac{2\pi ij}{N}}$, $z = \mathbf{0}$.

$$0 = \frac{1}{N}\sum_{k=1}^{N-1} F\left(2R\sin\left(\frac{\pi k}{N}\right)\right)\frac{1-exp(\frac{2\pi ik}{N})}{2R\sin(\frac{\pi k}{N})} + RH(R) \quad (5)$$

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Perturbation of the predator with fixed prey

Take $F(r) = r^{0.5} - r^2$, $H(r) = \frac{1}{r^2}$, $I(r) = \frac{1}{r^q+1}$ and add small perturbation to the predator, $z = \mathbf{0} + Be^{\lambda t}$.

Result

$$\lambda^2 + \lambda + \frac{N}{R^q+1} - \frac{NqR^q}{2(R^q+1)^2} = 0.$$

Solving the quadratic equation, we get

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1 + \frac{2N(qR^q - 2R^q - 2)}{(R^q+1)^2}}}{2}.$$

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Perturbation of both the ring with N prey particles and one predator

- For simplicity, we study the first order model.

$$\frac{dx_j}{dt} = \frac{1}{N}\sum_{k=0,k\neq j}^{N-1} F(\|x_j - x_k\|)\frac{x_j - x_k}{\|x_j - x_k\|} + H(\|x_j - z\|)(x - z), \qquad (6)$$

where x_j's are the positions of the preys and z is the position of predator.

- Taking $x_j = Re^{\frac{2\pi ij}{N}}(1 + \phi_j e^{\lambda t})$, $z = Be^{\lambda t}$, with $0 < \|\phi_j\|, \|B\| \ll 1$
- Taking $\phi_j = b_+ e^{\frac{2m\pi ij}{N}} + b_- e^{\frac{-2m\pi ij}{N}} + Be^{\frac{2\pi ij}{N}} + \bar{B}e^{\frac{-2\pi ij}{N}}$

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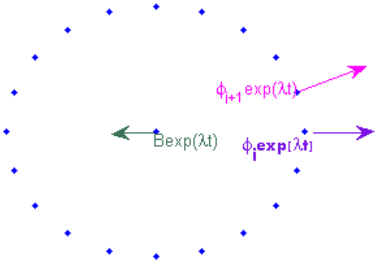


Result

The problem is reduced to an eigenvalue problem:

$$\lambda \begin{bmatrix} b_+ \\ b_- \\ B \\ \bar{B} \end{bmatrix} = \begin{bmatrix} I_1(m) & I_2(m) & 0 & 0 \\ I_2(m) & I_1(-m) & 0 & 0 \\ 0 & 0 & K_1 & K_2 \\ 0 & 0 & J_1 & J_2 \end{bmatrix} \begin{bmatrix} b_+ \\ b_- \\ B \\ \bar{B} \end{bmatrix} \tag{7}$$

where

$$I_1(m) = \frac{1}{N}\sum_{l=1}^{N} G_+\left(\frac{\pi l}{N}\right)\left(1 - e^{\frac{2(m+1)il\pi}{N}}\right) + \frac{H'(R)R}{2} + H(R),$$

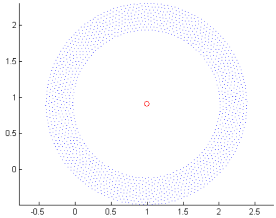
$$I_2(m) = \frac{1}{N}\sum_{l=1}^{N} G_-\left(\frac{\pi l}{N}\right)\left(e^{\frac{2mil\pi}{N}} - e^{\frac{2\pi il}{N}}\right) + \frac{H'(R)R}{2},$$

Result(continued)

$$
\begin{aligned}
K_1 =& \frac{4}{N}\sum_{l=1}^{N/2} G_+\left(\frac{\pi l}{N}\right)\sin^2\left(\frac{2\pi l}{N}\right) + RH'(R) + H(R),\\
K_2 =& -\frac{H'(R)}{2},\\
J_1 =& -\frac{H'(R)}{2} - \frac{H(R)}{R},\\
J_2 =& RH'(R) + H(R),\\
G_\pm(\theta) =& \frac{1}{2}\left[F'(2R|\sin(\theta)|) \pm \frac{F(2R|\sin(\theta)|)}{2R|\sin(\theta)|}\right].
\end{aligned}
$$

Constant density within the annulus at the steady state of first order model in 2D

- **Numerical result**: taking 1500 particles



- **Analytic result**: Let $D \subset \mathbb{R}^2$ be the annulus whose inner radius and outer radius are r and R, respectively. The system has a steady state for which $\rho(x)$ is constant inside D and is 0 outside D under the condition that either the interaction of predator-prey $H(r) = \frac{1}{r^2}$ or $H(r)$ is constant.

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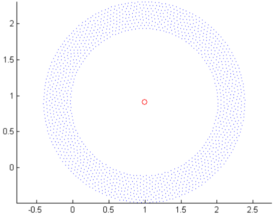

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For simplicity, we take $f(r) = F(r)/r$, where $F(r) = \frac{1}{r} - r$. The continuum form of the system is:

$$\rho_t(x,t) + \nabla_x \cdot (v(x)\rho(x,t)) = 0 \tag{8}$$

$$\begin{aligned} v(x) &= \int_{\mathbb{R}^2} f(\|x-y\|)(x-y)\rho(y)dy + H(\|x-z\|)(x-z) \\ &= \int_{\mathbb{R}^2} [\nabla_x ln\|x-y\| - Id_2(x-y)]\rho(y)dy + H(\|x-z\|)(x-z), \end{aligned} \tag{9}$$

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For simplicity, we take $f(r) = F(r)/r$, where $F(r) = \frac{1}{r} - r$. The continuum form of the system is:

$$\rho_t(x,t) + \bigtriangledown_x \cdot (v(x)\rho(x,t)) = 0 \tag{8}$$

$$\begin{aligned} v(x) &= \int_{\mathbb{R}^2} f(\|x-y\|)(x-y)\rho(y)dy + H(\|x-z\|)(x-z) \\ &= \int_{\mathbb{R}^2} [\bigtriangledown_x ln\|x-y\| - Id_2(x-y)]\rho(y)dy + H(\|x-z\|)(x-z), \end{aligned} \tag{9}$$

where Id_2 denotes the 2×2 identity matrix and $\rho(x,t)$ denotes the density of the prey particles.

Sketch of the proof

- Use method of characteristics:

$$\frac{dx}{dt} = v; \frac{d\rho}{dt} = -(\bigtriangledown \cdot v)\rho.$$

- Thus, $-\frac{d\rho}{dt} = -(2\pi\rho - 2M + \bigtriangledown_x \cdot [H(\|x-z\|)(x-z)])\rho$, where $M = \int_{\mathbb{R}^2} \rho(y)dy$ is the conserved mass.
- To make ρ independent of the position inside the domain, $\bigtriangledown_x \cdot [H(\|x-z\|)(x-z)]$ has to be constant, hence $H(r) = \frac{1}{r^2}$ or constant.
- $v(x) = \pi\rho(t)x(1 - \frac{r^2}{\|x\|^2}) - x|D(t)|\rho(t) + H(\|x-z\|)(x-z)$
- At the boundary of the annulus, both $v(x)$ and $\bigtriangledown \cdot v$ are 0. Using this fact, we get

$$R^2 - r^2 = 1;$$

$$\pi\rho(t)\left(1 - \frac{r^2}{\|x\|^2}\right) - |D(t)|\rho(t) + H(\|x\|) = 0.$$

- When x is on the inner boundary, $\|x\| = r$, solving the above system gives $r = 1, R = \sqrt{2}$

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- Fine tuning the collision modeling of the bounded room model;
- Further reasearch into the effects of obstacles on peoples exit speed;
- Evolution study of the prey swarming interacted with the predator;
- Understand the optimal path for predators to appraoch the prey.

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