Chaparral Populations Models in Response to Wildfires

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Abstract

We present three nonlinear systems of difference equations that model three chaparral plant types in response to wildfire: non-sprouters (NS), facultative sprouters (FS), and obligate sprouters (OS). A preliminary design for a spatial model is also presented. Population parameters in the discrete, temporal system (e.g. seed germination, seed production, and seed predation) are implemented to act upon the aggregate populations whereas individual plant behaviors (e.g. crown size) are considered in the spatial model. When the recurrence of wildfire is more frequent, NS plant types experience localized extinction whereas FS plant types are better adapted and will perpetuate much further in time. Conversely, an infrequent recurrence of wildfire will result in greater population survival for all plant types. We explore how the long term population behavior depends on seed release rate, seed germination rate, seed predation rate and seedling survivorship.

1 Introduction

The Mediterranean-type ecological habitat of the Santa Monica Mountains is known to be one of the most fire-prone environments in the world. The Santa Monica Mountains are encircled by the Pacific Ocean and Greater Los Angeles; the region is inhabited by a large number of communities including households, businesses, and institutions of higher-learning. The landscape of the Santa Monica Mountains is defined by a plethora of plant species with chaparral shrubs being the dominant plant type as they account for over 50 percent of the region's plant life [12]. In recent years, the region has experienced an increase in the frequency of wildfire which has been a substantial

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economic burden to the residing communities and a stress on the surrounding wildlife. Much of the region's topography includes sloping hills, and as "most chaparral grows on...mountains where the steep slopes range from 25 to 70 [degrees]" [7], materials including buildings, rocks, and soil that are not held firmly in place by plant matter often slide and fall, or at the very least experience weakening in their foundations [1].

While chaparral plant species are known to be fire-adapted, the increased fire frequency plaguing their habitat has the capacity to render particular species extinct, at least in a localized sense. Such extinction would alter the region's landscape and its viability for inhabitance as the lack of chaparral would undoubtedly lead to an increase in rock and mud slides, floods, and an overall atrophy of structural foundations [7]. In view of this, it is of great interest to model the behaviors of chaparral plant types in response to wildfire as their survival has far-reaching consequences.

1.1 Chaparral Responses to Wildfire

Chaparral plant species survive the incidence of wildfire by way of three singular life history types: non-sprouters (NS), facultative sprouters (FS), and obligate sprouters (OS). NS plant types are decimated by wildfire and are sustained solely by the germination of fire-stimulated seeds. [6] If the seeds are located such that they receive ample sun, [6] specifically in open canopy areas, they develop into seedlings. Following approximately six years [14], seedlings assume full maturity and the ability to drop seeds; thereby, the NS plant type is again equipped to endure wildfire. However, given that the subsistence of the NS plant type is dependent upon the presence of seeds in the soil, a rapid sucession of fires may result in the localized extinction of NS plant types as seedlings are deprived of the time needed to mature and release seeds. The population cycle of the NS plant type is depicted in Figure 1.1

FS plant types are also burned by wildfire, yet they survive through a combination of post-fire seed germination and vegetative resprouting. Seeds are stimulated by fire and germinate into seedlings under partial shade. If fire is absent for roughly six years thereafter–as with NS plant types–seedlings will develop into mature plants [14]. Less than 10% of the seedlings survive the six year maturation period. Alternatively, resprouts come from the burned root crowns of mature plants and prior resprouts post-fire. Unlike seedlings, they begin to drop seeds one year post-fire, and unlike mature plants, their rate of seed dispersal grows until the resprout fully recovers and its release rate hits the same as that of mature plants [2]. Mature plants release seeds at a set rate and will only be killed by fire. FS plant types are best suited to reproduce after wildfire because they have a dual survival response.

In contrast to NS and FS plant types, OS plant types do not persist by way of post-fire seed germination. Instead, OS plant types endure solely by means of vegetative resprouting [6]. Resprouts grow out of the burned root crowns of both pre-fire mature plants and pre-fire resprouts. They are assumed to undergo a logistic decay, and will start to drop seeds one year post-fire with a growing releasing rate analogous to that of FS resprouts. It takes two to three years for resprouts to fully recover from fire, after which their behavior parallels mature plants. Mature plants are only killed by fire, and they release seeds at a set rate throughout their lifespan. If fire is too recurrent, localized extinction for OS plant types is inevitable as they will be unable to establish mature plants or an adequate number of resprouts to release and grow the required seedbank. The population cycles of the FS and OS plant type are depicted in Figure 1.2.

From 1925-2001, the average fire frequency for the entire Santa Monica Mountains was 32 years [12]. For our particular study site in Malibu, the average fire frequency from 1985-2011 has been just over 6 years. Already *Ceanothus megacarpus* are experiencing localized extinction in this site [11]. Given that the surrounding community, including Pepperdine University, is built upon a landscape that is greatly supported by the presence of chaparral shrubs, the development of predictive models describing the responses of the aforementioned chaparral plant types to the incidence of wildfire is of great importance.

2 NS Model

2.1 Assumptions

As with each plant type, the NS model assumes that all events and processes occur on an annual basis; therefore certain events (e.g. germination) will appear to be instantaneous. The initializing event will always be the incidence of a wildfire which will simultaneously annihilate the NS population as well as germinate seeds. The recurrence of wildfire will be assumed to follow a Poisson process with expected value λ. Seeds, S_t, are assumed to be eaten by predators at a given rate, ρ, as well as germinated at a given rate g. The germinated seeds will become seedlings, Y_t, and these seedlings will undergo logistic decay with a carrying capacity proportional to their initial post-fire population. Using nonlinear regression analysis, we fit measured data points of seedling survival to Morishita's discrete logistic curve outlined in [8]. The seedlings follow their logistic decay path until they are either killed by wildfire, F_t, or promoted to mature plants, P_t. The time post-fire until seedling maturation, m, is approximately six years,

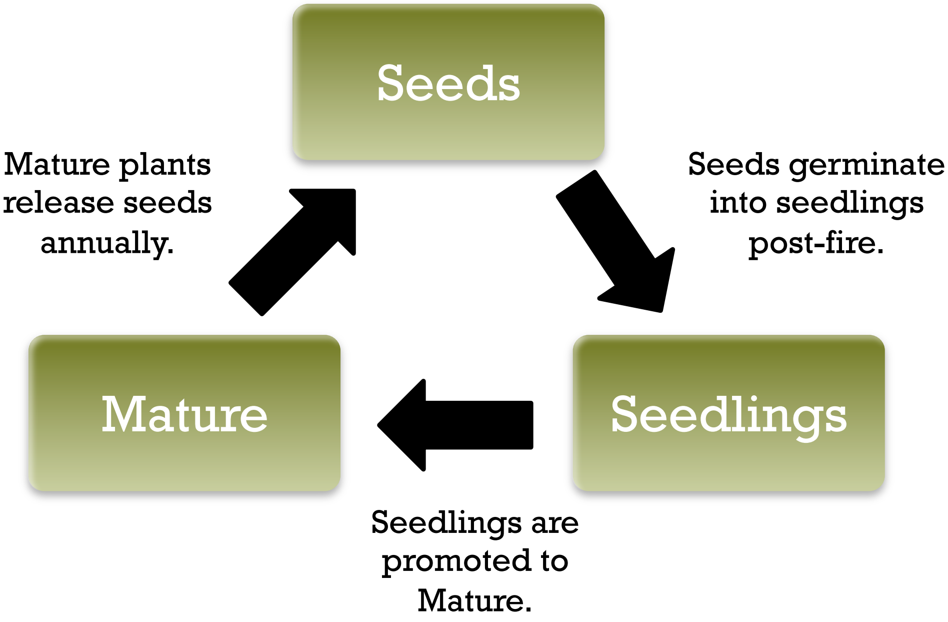

Figure 1.1: NS Population Cycle

but can be modeled as $m \sim \text{Poisson}\,(6)$. Mature plants will release seeds at a given rate r and will only be eliminated in the occurrence of a wildfire.

2.2 Equations

For the NS plant type, the model is as follows:

$$\begin{aligned} Y_t &= (1-P_t)\left[Y^* + \alpha Y_t\left(1 - \frac{Y^*}{K_t}\right)\right] \\ M_t &= (1-F_t)M_{t-1} + P_t Y_{t-1} \\ S_t &= [(1-F_t)S_{t-1} + rM_t](1-\rho). \end{aligned} \tag{1}$$

where,

$$Y^* = (1-F_t)Y_{t-1} + F_t g S_{t-1} \tag{2}$$

Both F_t and P_t are indicator functions with value whenever fire or seedling maturation occur, respectively. It should be noted, K_t, the seedling carrying capacity is also contingent on the incidence of wildfire as its value is always a fraction of the number of seeds germinated post-fire (4). The fractional constant, k, that reduces K_t is the estimated carrying capacity, $\hat{K_0}$, of Morishita's equation [8] divided by the initial data point (number of seedlings), Y_0, used in the nonlinear regression (3).

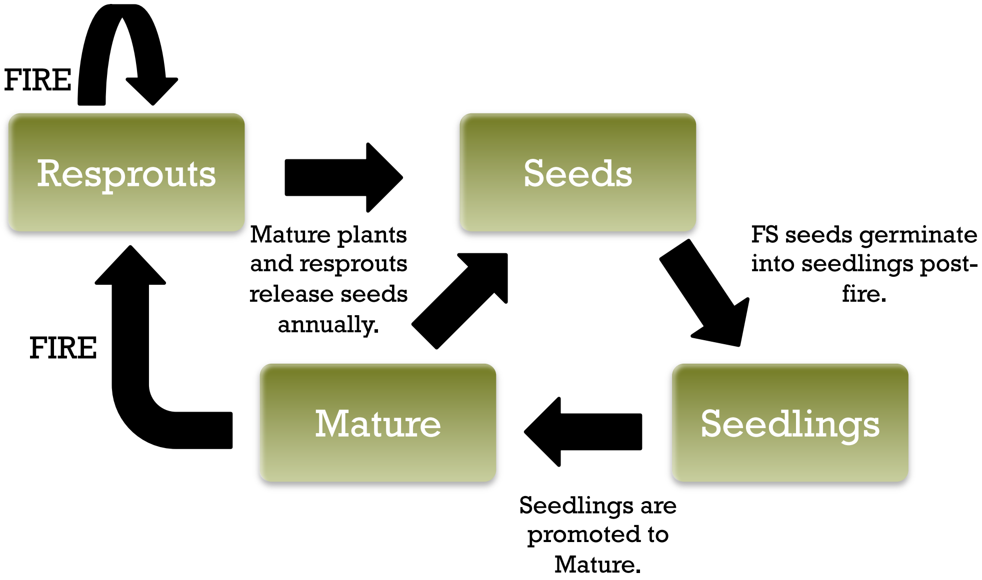

Figure 1.2: FS/OS Population Cycle

$$k = \frac{\hat{K_0}}{Y_0} \tag{3}$$

In essence this assumes that at all points in time, seedlings are decaying to a constant fraction of the most recently germinated seedbank. Thus,

$$K_t = (1 - F_t)K_{t-1} + F_t g S_{t-1} k \tag{4}$$

Seedling logistic decay data is based upon *Ceanothus megacarpus* post-fire data gathered by Dr. Stephen Davis and is fit according to the continuous logistic equation by minimizing the sum of the residuals squared, then the curve is discretized on an annual basis. (See Figure 2.1) The initial number of seedlings recorded post-fire, Y_0, in this particular data set was 238, and since $\hat{K_0}$ was found to be approximately 53.79, $k \approx 0.226$. The parameter value $\hat{\alpha}$ was found to be approximately 0.239. These values of k and $\hat{\alpha}$ are used for all NS simulations throughout the remainder of this paper.

2.3 Results

The NS Model (1) is tested across a variety of scenarios; by varying particular parameters, population behavior in response to wildfire differs. These responses are intuitive: higher seed germination and seed release rates should lead to a more intransigent population even in the event of fire; predation rates will undoubtedly alter the size of the

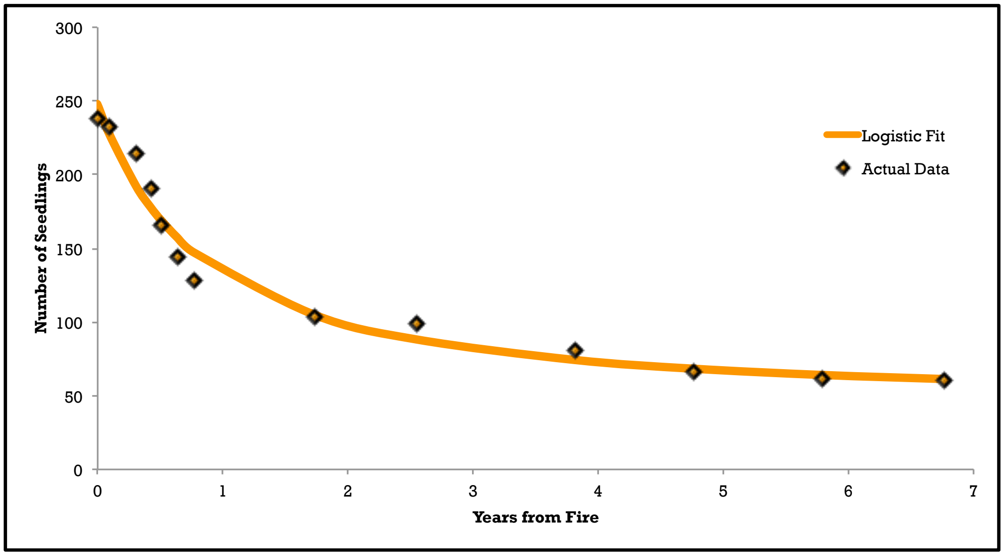

Figure 2.1: Logistic Decay Fit for *Ceanothus megacarpus* Data

seedbank, and thus vary the size of the initial seedling population post-fire. For the purposes of reflecting the actual behaviors of NS plant types, parameters can be fixed at particular values for specific NS species; however, data on these values vary among sources (e.g. [5, 4, 14, 13, 9, 10]) and so sample values were selected among these ranges. When these values are fixed, the actual impact of fire frequency can be observed. It is evident if the years to maturation and the fire frequency are of the same value, an immediate annihilation of the population will occur after the second fire Figure 2.2. This is reasonable as the seedling population is promoted in the same year as the next fire disallowing the newly promoted mature plants ample time to release seeds into a seedbank. When the seedling maturation year is fixed at six and the fire frequency is fixed at a moderate rate (e.g. 12 years) the NS population is seen to persist longer, yet its decay is evident, Figure 2.3. A low fire frequency (e.g. 20 years), allows the population to perpetuate much further in time, Figure 2.4. Both a moderate rate and a low rate of fire frequency allot NS plant types sufficient time to accumulate a sizable seedbank, and therefore a robust seedling population post-fire.

If the occurrence of both fire and seedling promotion are modeled as Poisson processes with particular means, the long term behavior of the population is better understood when observing an average over many simulations. When the mean fire frequency is low (20 years), the NS population can be seen to sustain itself through multiple fires, Figure 2.5; however, a high mean fire frequency (6 years) will lead a population into extinction, Figure 2.6. Note that when the maturation year and the fire frequency

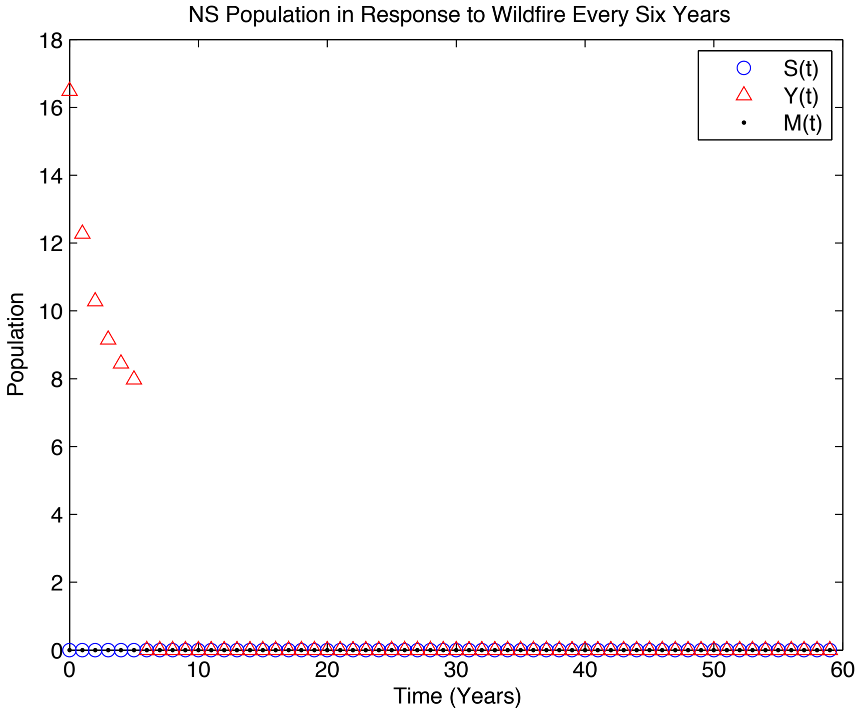

Figure 2.2: Maturation Rate of Six Years, Fire Frequency of Six Years

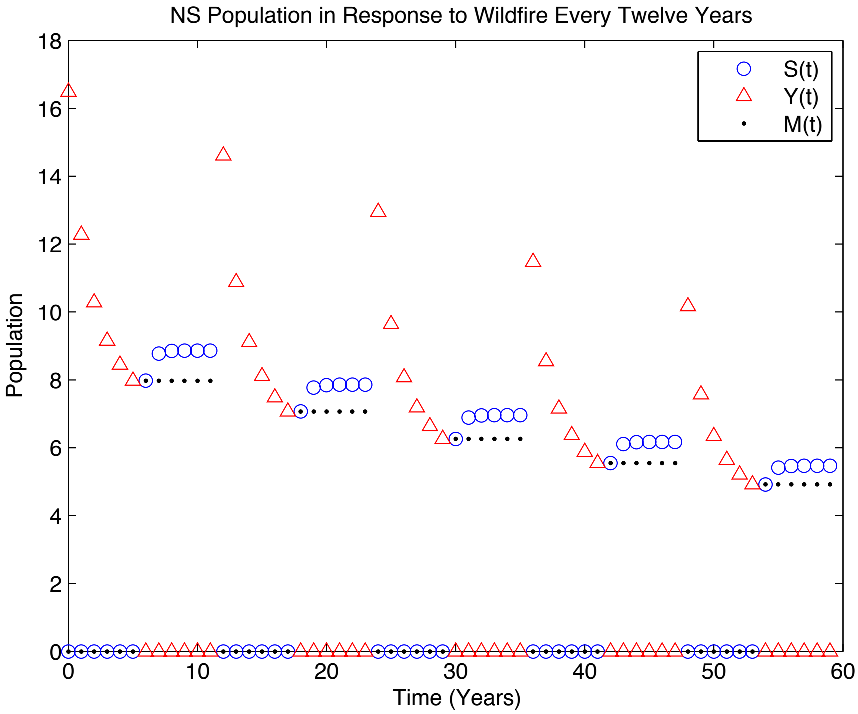

Figure 2.3: Maturation Rate of Six Years, Fire Frequency of 12 Years

are both fixed at six years, the population is annihilated immediately after the second fire, but when the maturation year and the fire frequency are considered random

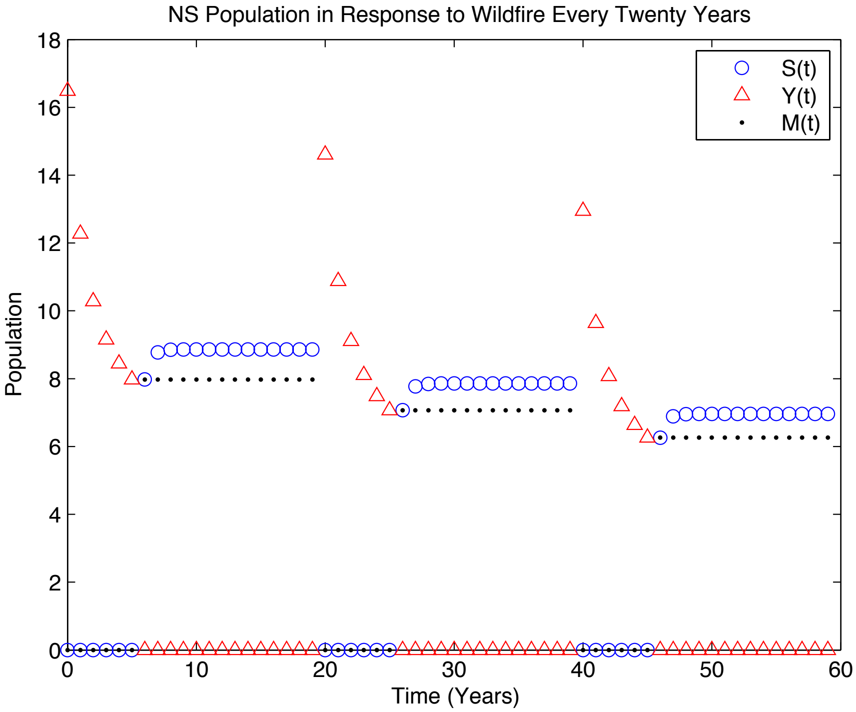

Figure 2.4: Maturation Rate of Six Years, Fire Frequency of 20 Years

events, on average the population will persist slightly longer since the probability that the maturation year will be less than the fire frequency is about 0.24. For the same mean maturation year and expected fire frequencies of 12 and 20 years, this probability jumps to about 0.58 and 0.60 respectively. The multiplicity of outcomes observed with respect to variation in both the model's parameters and fire frequency necessitates an exploration of parameter relationships.

It is of great interest to find parameter combinations that produce mature populations remaining in some sort of equilibrium. In the absence of fire, seedlings may eventually reach their carrying capacity. If this occurs, then after promotion, the mature population will also be equal to the carrying capacity of the seedlings. If this mature population is to remain in equilibrium, then the seedling carrying capacity must be the limiting value of the mature population. In this case the seedbank may approach a constant limit. It cannot be known whether the constant limit actually exists, however, if it does then the following relationship is true:

$$\bar{S} = (1-\rho)\bar{S} + r\bar{M} \tag{5}$$

Thus the constant value of the seedbank is:

$$\bar{S} = \frac{r(1-\rho)}{\rho}\bar{M} \tag{6}$$

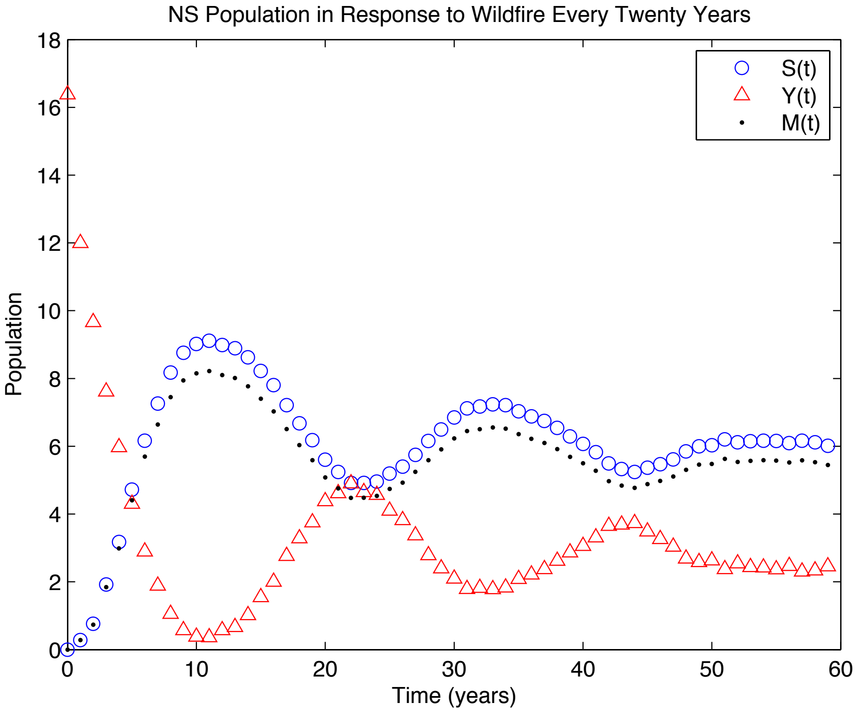

Figure 2.5: Expected Maturation Rate of Six Years, Expected Fire Frequency of 20 Years

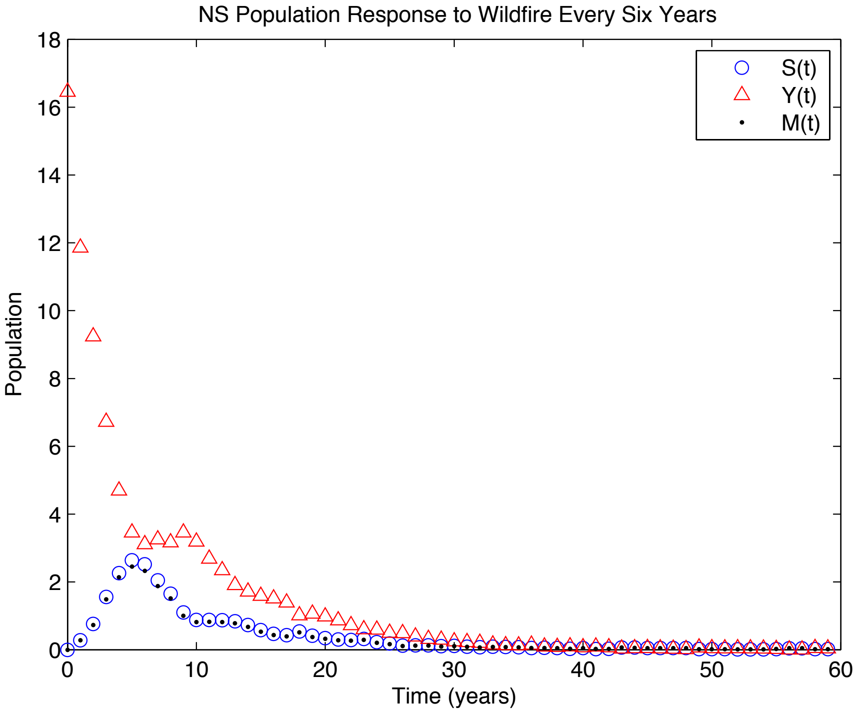

Figure 2.6: Expected Maturation Rate of Six Years, Expected Fire Frequency of 6 Years

Since the equilibrium mature population is in equilibrium, its limiting value is the seedling carrying capacity, and the following relationship is noticed:

$$\begin{aligned}\bar{M} &= K \\ &= g\bar{S}k \\ &= \frac{grk(1-\rho)}{\rho}\bar{M}\end{aligned} \tag{7}$$

Therefore the mature population is in equilibrium when:

$$grk\left(\frac{1-\rho}{\rho}\right) = 1 \tag{8}$$

To test this result experimentally, we varied the parameters and plotted the values corresponding to equilibrium mature populations. Figure 2.7 depicts the results for 770 million tested parameter combinations: release rates were varied, germination rates were varied, predation rates were varied, and fire frequencies were varied. Notice that a change in fire frequency did not have any bearing on the parameter relationship. The values of k and α in the seedling logistic decay were not varied initially.

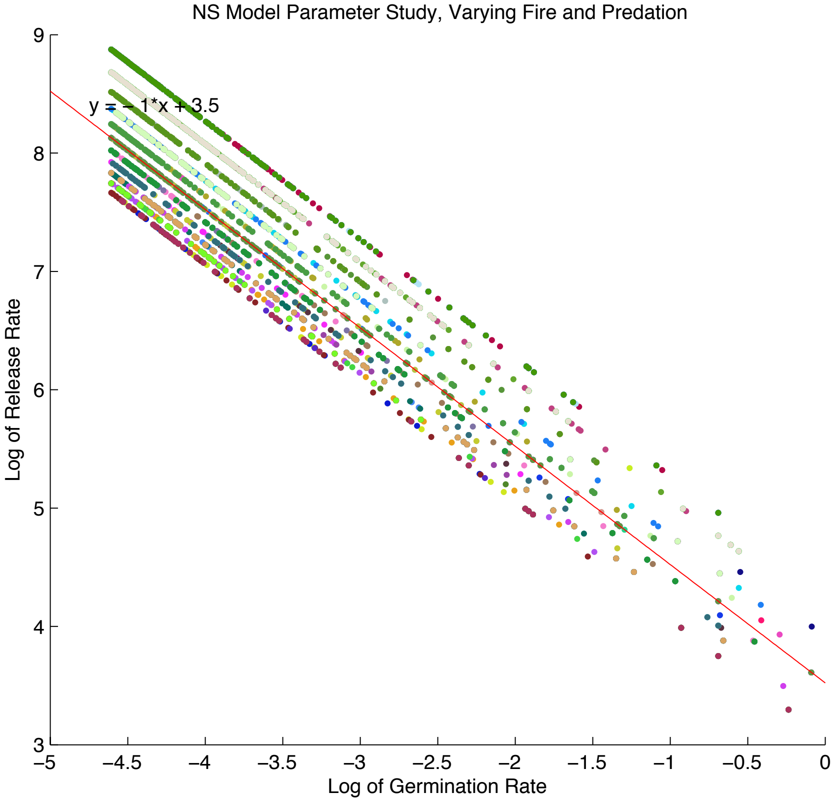

Figure 2.7: Parameter Relationships Across 770 Million Combinations

Fire was then held constant, and k and α were varied along with the other parameters, Figure 2.8. Notice that the relationship holds when α was greater than or equal to α from the logistic fit. However, when α is less than the fitted value, concavity arises. Since α controls how fast seedlings approach their carrying capacity, when α is small, seedlings are unlikely to reach their capacity value, and thus the mature population is not limited and will fluctuate.

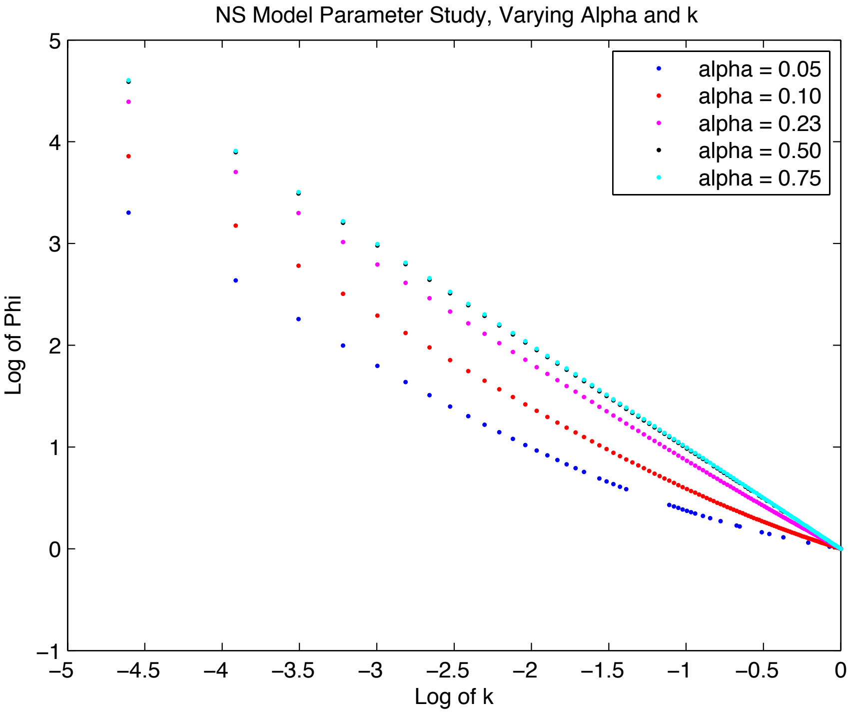

Figure 2.8: Parameter Relationships Varying α and k

Experimentation found the mature population equilibrium to be unstable. When the product of the parameters is greater than one, the mature population will grow without bound, Figure 2.10. Conversely, the mature population will decay to zero if the product of the parameters is less than one, Figure 2.11. Of course, this can only be true if fire does not occur at a rate such that seedlings are unable to promote.

In actuality, the equilibrium is unlikely to ever be reached due to the fact that parameter values do have variance in nature and other natural phenomena such as competition amongst plants, disease, and drought are not accounted for thus far. The model does very well qualitatively in predicting the behavior of the *Ceanothus megacarpus* (Cms) plant type as evidenced in Figure 2.12.

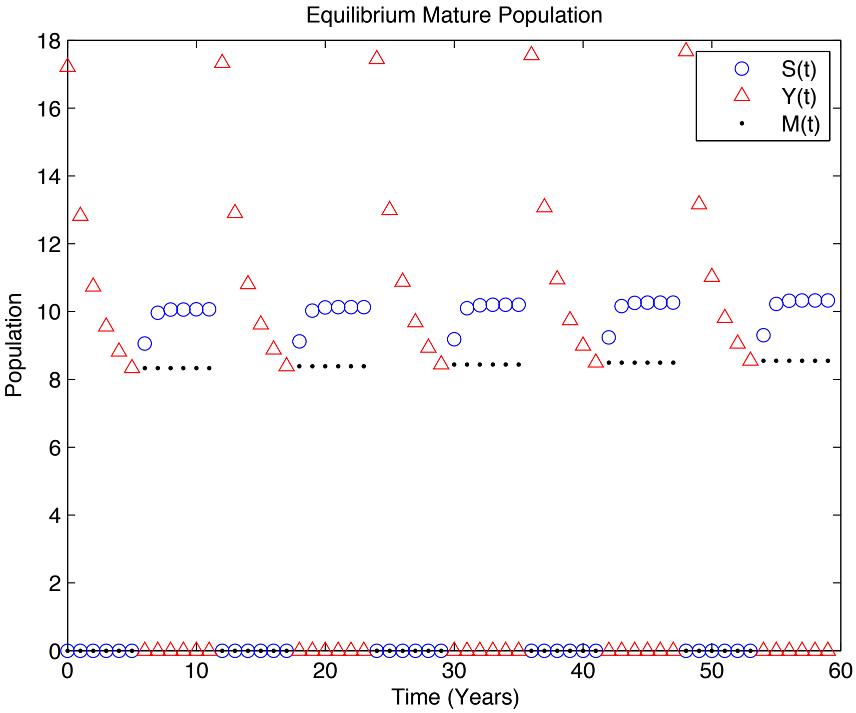

Figure 2.9: Product of the Parameters Equal to One

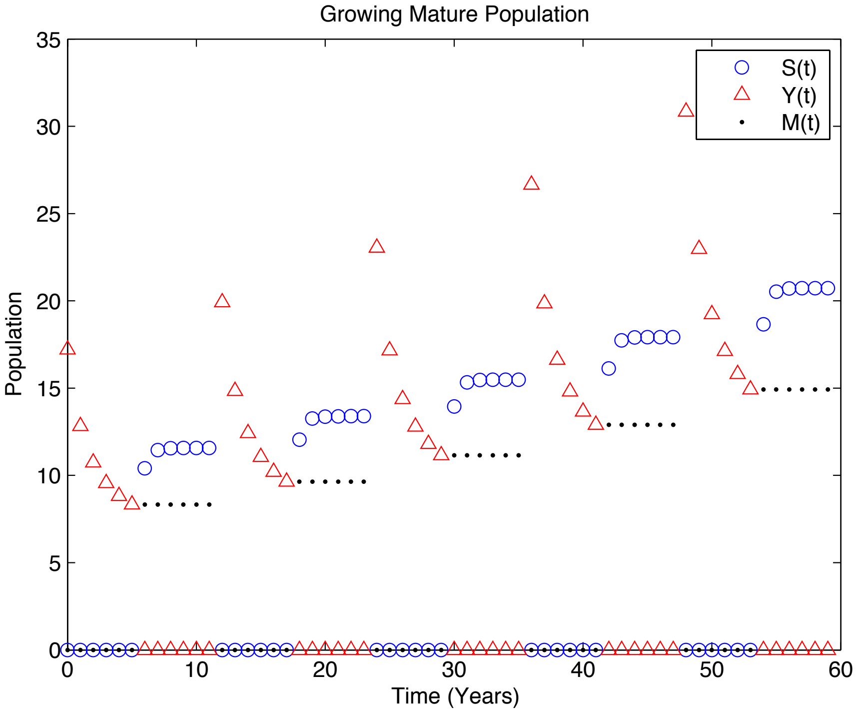

Figure 2.10: Product of the Parameters Greater Than One

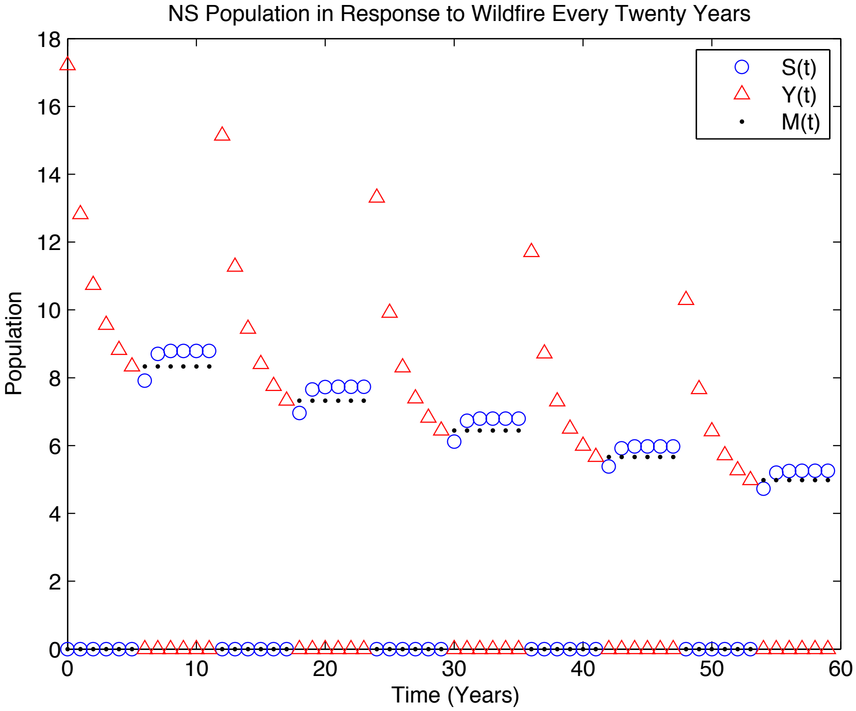

Figure 2.11: Product of the Parameters Less Than One

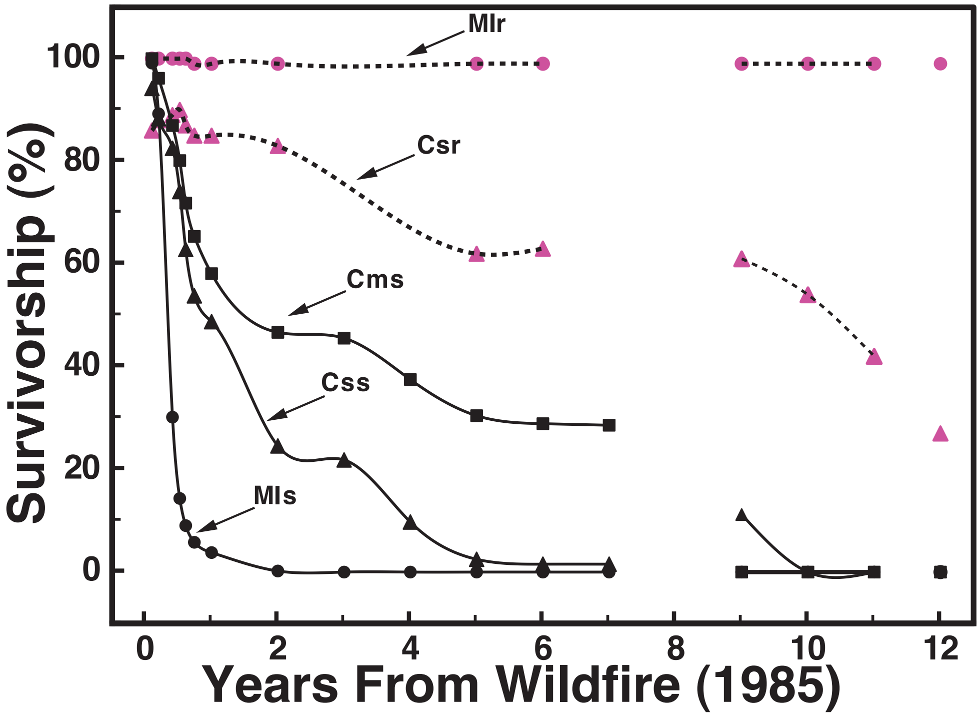

Figure 2.12: NS and FS Seedling and Resprout Survival from 1985-1997

3 OS Model

3.1 Assumptions

Fire, F_t, and promotion, P_t are modeled the same as in the NS model. Unlike the NS plant type, seeds, S_t, are assumed to be killed off by fire and recruit when dispersed by resprouts and mature plants. A certain percentage, ρ, of seeds are assumed to be predated by animals before they can germinate. After germination, seeds become seedlings, Y_t, that undergo a logistic decay with a changing carrying capacity, K_Y. When seedlings reach their maturation year, m, without the occurence of fire coming prior, they are promoted, P_t, into mature plants, M_t. Resprouts, R_t, grow from burned root crowns post-fire and die logistically analogous to seedlings with a changing carrying capacity, K_R. They release seeds one year post-fire but at a relatively low initial rate, r_t. This rate grows over time until resprouts are fully recovered and their behavior parallels that of mature plants. Mature plants are only eliminated in the presence of fire; otherwise, they release seeds at a set rate, r, which is also the rate to which the releasing rate of resprouts is growing.

3.2 Equations

$$\begin{aligned} Y_t &= (1-F_t)(1-P_t)\left[Y_{t-1} + gS_{t-1} + \alpha_Y Y_t\left(1 - \frac{Y_{t-1}+gS_{t-1}}{K_Y}\right)\right] \\ R_t &= \left[R_{t-1} + F_t M_{t-1} + \alpha_R R_t\left(1 - \frac{R_{t-1}+F_t M_{t-1}}{K_R}\right)\right] \\ M_t &= (1-F_t)M_{t-1} + P_t Y_{t-1} \\ S_t &= (1-F_t)(S_{t-1} + rM_t + r_t R_t)(1-\rho)(1-g) \end{aligned} \tag{9}$$

Due to the scarcity of data in regards to OS plant types, feasible parameter values are unobtainable and thus we have omitted results.

4 FS Model

4.1 Assumptions

The FS model incorporates ideas from both the NS and OS models. Fire, F_t, and promotion, P_t are modeled the same as in the NS model. Right after fire, FS seeds, S_t, are germinated at a certain rate, g (different than that of the NS plant type). After germination, seeds develop into seedlings, Y_t, and begin to decay logistically with a carrying capacity, K_Y, proportional to their initial population post-fire. They will be promoted to mature plants, M_t, after the maturation year, m, which is approximately six years–this can be modeled as a Poisson process–provided that no fire comes in between. Mature plants release seeds at a set rate, r, also building up a seedbank. Resprouts, R_t, vegetate immediately post-fire from the root crowns underground. They begin to release seeds one year post-fire but at a significantly lower rate than that of mature plants; however, they recover in two to three years after, and in the interim their releasing rate, r_t, grows to that of mature plants. Resprouts likewise undergo a logistic decay with a carrying capacity, K_R, proportional to their initial population post-fire.

4.2 Equations

For the FS plant type, the model is as follows:

$$\begin{aligned} Y_t &= (1-P_t)\left[Y^* + \alpha_Y Y_t\left(1-\frac{Y^*}{K_Y}\right)\right] \\ R_t &= R_{t-1} + F_t M_{t-1} + \alpha_R R_t\left(1-\frac{R_{t-1}+F_t M_{t-1}}{K_R}\right) \\ M_t &= (1-F_t)M_{t-1} + P_t Y_{t-1} \\ S_t &= (1-F_t)(S_{t-1} + rM_t + r_t R_t)(1-\rho). \end{aligned} \tag{10}$$

where,

$$Y^* = (1-F_t)Y_{t-1} + F_t g S_{t-1} \tag{11}$$

Both carrying capacities, K_Y and K_R, are proportional to their initial populations post-fire: the number of seeds germinated and the number of total mature plants and resprouts pre-fire, repectively. The fractions that generate new carrying capacities each time fire occurs, k_Y and k_R, are obtained by dividing the estimated carrying capacities $\hat{K_{Y_0}}$ and $\hat{K_{R_0}}$ from the fit of Morishita's equation [8] by the initial population–Y_0 and R_0, repsectively–at the time of the last fire used in the nonlinear regression. Therefore,

$$k_Y = \frac{\hat{K_{Y_0}}}{Y_0} \tag{12}$$

and,

$$k_R = \frac{\hat{K_{R_0}}}{R_0} \tag{13}$$

It is assumed in the above equations that the seedlings are decaying to a fraction, k_Y, of the most recently germinated seedbank, and resprouts are decaying to a fraction, k_R, of the most recently resprouted plants. Thus,

$$K_Y = (1-F_t)K_{Y_{t-1}} + F_t g S_{t-1} k_Y \tag{14}$$

and,

$$K_R = (1 - F_t)K_{R_{t-1}} + F_t(M_{t-1} + R_{t-1})k_R \tag{15}$$

Logistic decays are fit in the same manner as in the NS model. The initial number of seedlings recorded post-fire in the data set for *Ceanothus spinosus* was 446, and since $\hat{K_{Y_0}}$ was found to be approximately 49.7, $k_Y \approx 0.1114$. The parameter value $\hat{\alpha_Y}$ was found to be approximately 0.157. The initial number of resprouts recorded pre-fire in this particular data set was 83, and $\hat{K_{R_0}}$ was found to be approximately 0.38. The parameter value $\hat{\alpha_R}$ was found to be approximately 0.0005. (See Figure 4.1) The values, k_Y, $\hat{\alpha_Y}$, k_R, and $\hat{\alpha_R}$ are used for all FS simulations throughout the remainder of this paper.

Explorations are also made using values of k_Y, $\hat{\alpha_Y}$, k_R, and $\hat{\alpha_R}$, for *Malosma laurina*. *Malosma laurina* resprouts have an exorbitantly high survival rate of 99 percent, but their seedlings are much weaker with a survival rate close to zero. (See Figure 4.2) This particular FS species is studied generally as an extreme case, otherwise the values for *Ceanothus spinosus* are used.

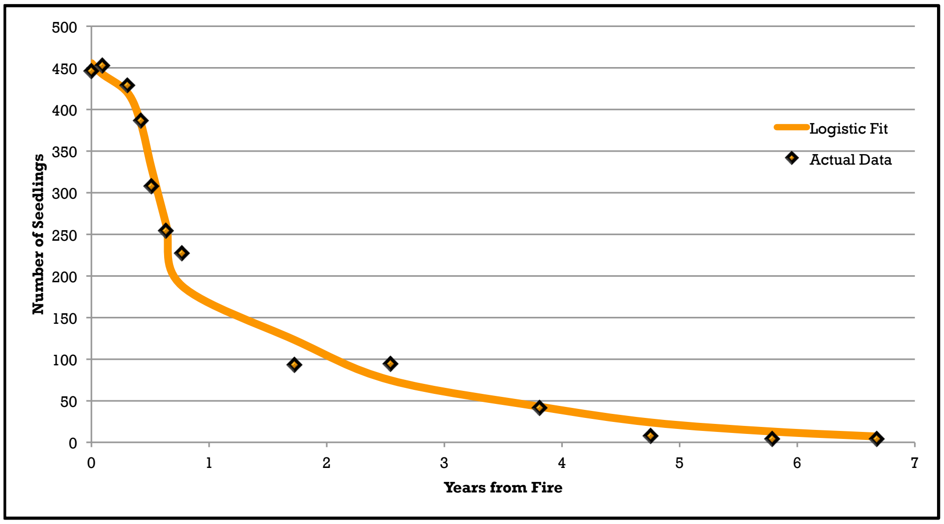

Figure 4.1: C.S. Seedlings Logistic Fit

4.3 Results

We tested rhe FS Model, (10), using a number of different parameters. By varying those parameters, the behavior of the model can be totally different. With a moderate germination rate [3] and release rate [5], the model is very stable at a set fire frequency; the predation rate changes the size of the seedbank, thus resulting in the change of the

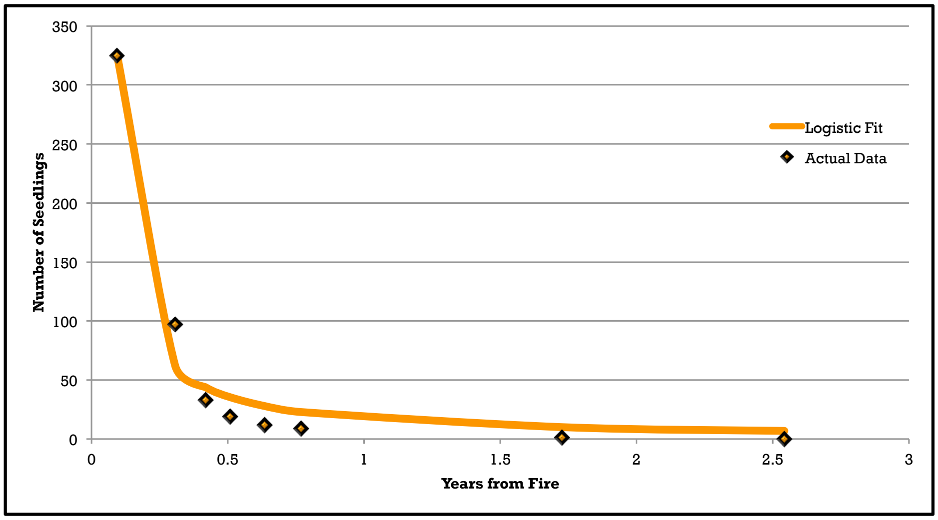

Figure 4.2: M.L. Seedlings Logistic Fit

seedling population. When the maturation year and the fire frequency are the same, serious damage will happen to the population but not enough to destroy the whole species because of resprouts' existence. If fire comes even sooner before resprouts could release enough seeds, the entire population will be wiped out. (See Figures 4.3 - 4.9.)

In the absence of fire, seedlings will eventually promote into a mature population that remains constant, and resprouts will fully recover and stop the decaying process. In this case the seedbank can be seen to approach a limit:

$$S = (1-\rho)[S + r(M+R)] \tag{16}$$

Without the occurence of fire, the number of mature plants and resprouts is a constant as resprouts stop decaying. Therefore, the total population of mature plants and resrprouts is set aside so that only the parameters that represent the influence of surroundings are left. For the purpose of easy reference and testing, a new parameter ϕ is introduced in as follows:

$$\phi = \frac{gr(1-\rho)}{\rho}. \tag{17}$$

In the occurence of fire, the seedbank, which is a constant now, will germinate into a new seedling population.

$$gS = g\frac{r(M+R)(1-\rho)}{\rho} = Y_{FIRE} \tag{18}$$

Since mature plants are grown from the promoted seedlings, and resprouts and seedlings both decay logistically before maturation, the key factors that affect the total population of M and R are the two carrying capacities K_Y and K_R. To go a step further,

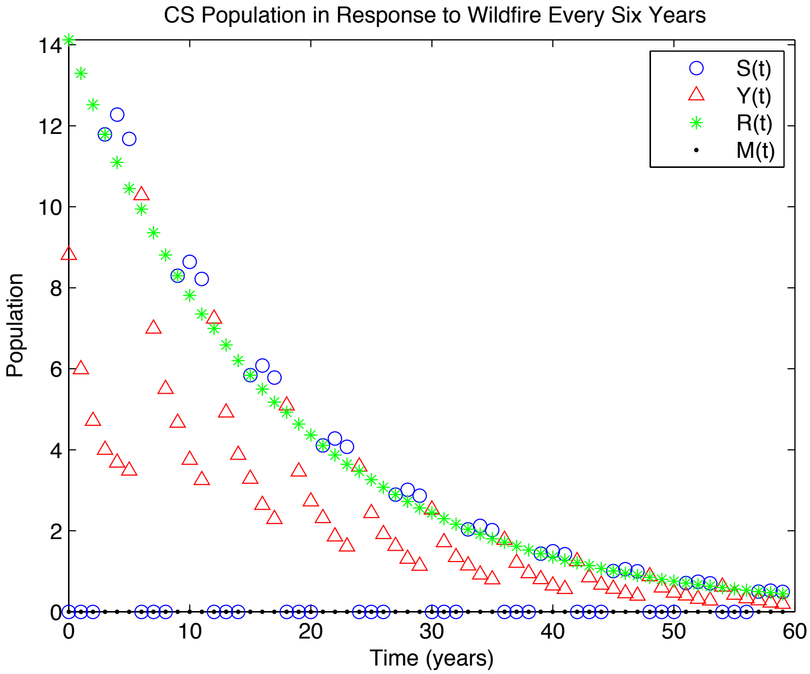

Figure 4.3: C.S. Typical Fire Frequency, Typical Germination Rate

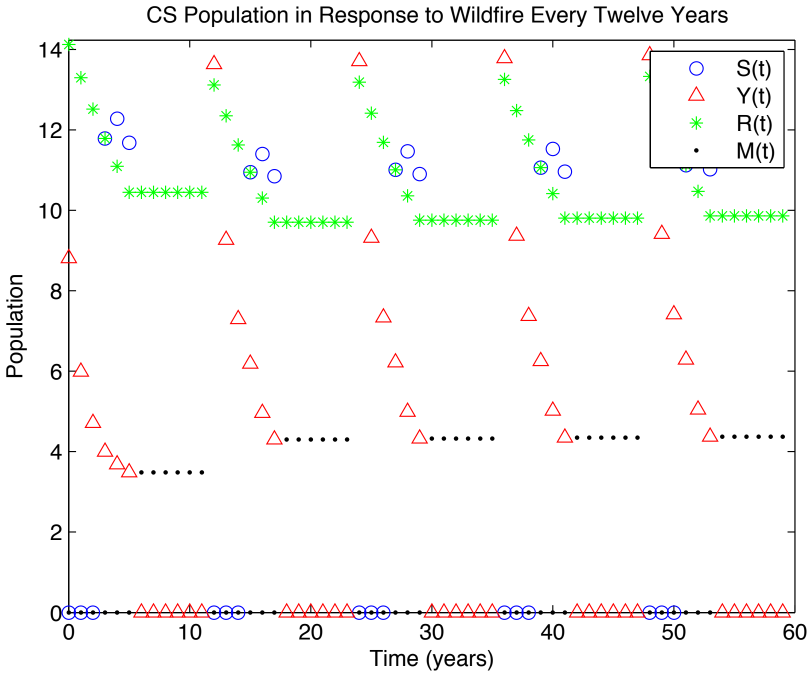

Figure 4.4: C.S. High Fire Frequency, Typical Germination Rate

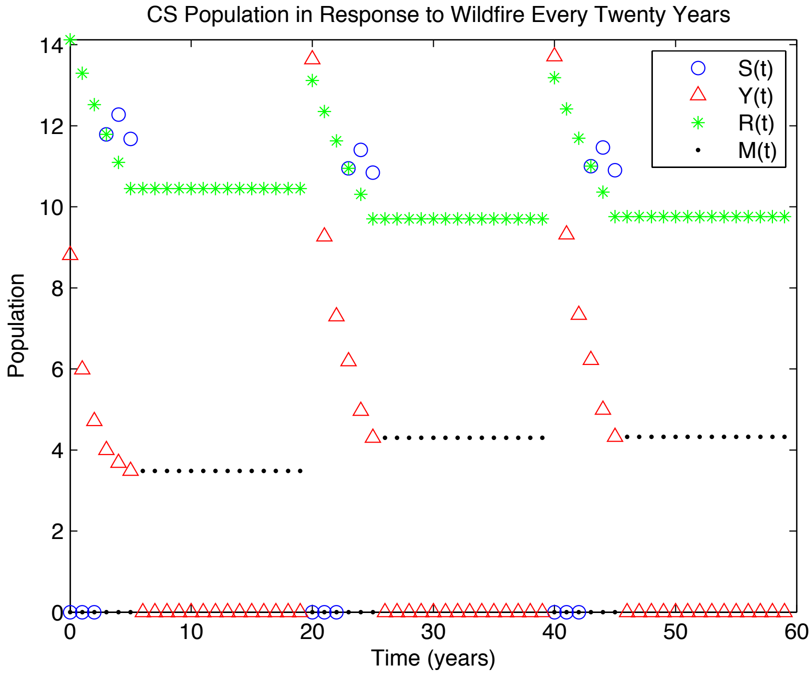

Figure 4.5: C.S. Low Fire Frequency, Typical Germination Rate

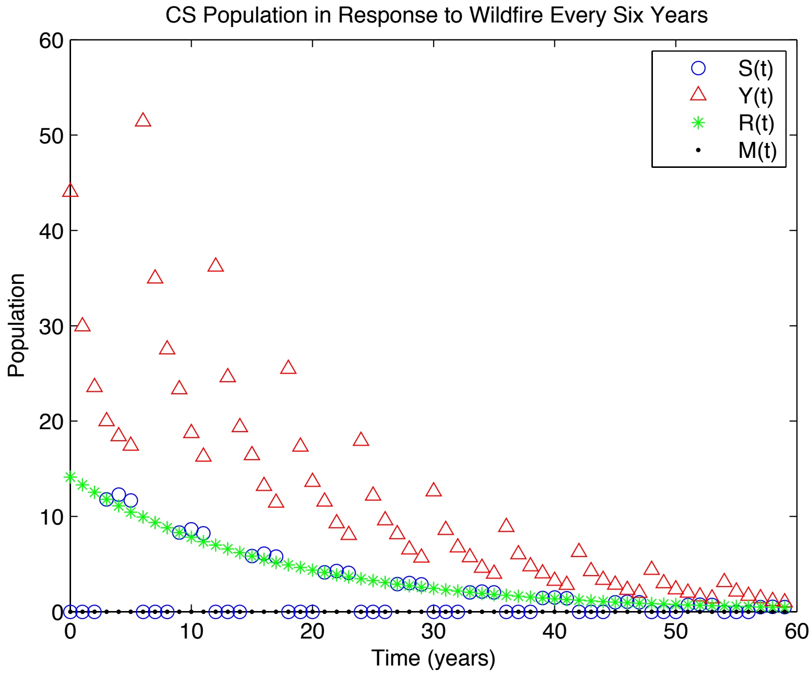

Figure 4.6: C.S. High Fire Frequency, High Germination Rate

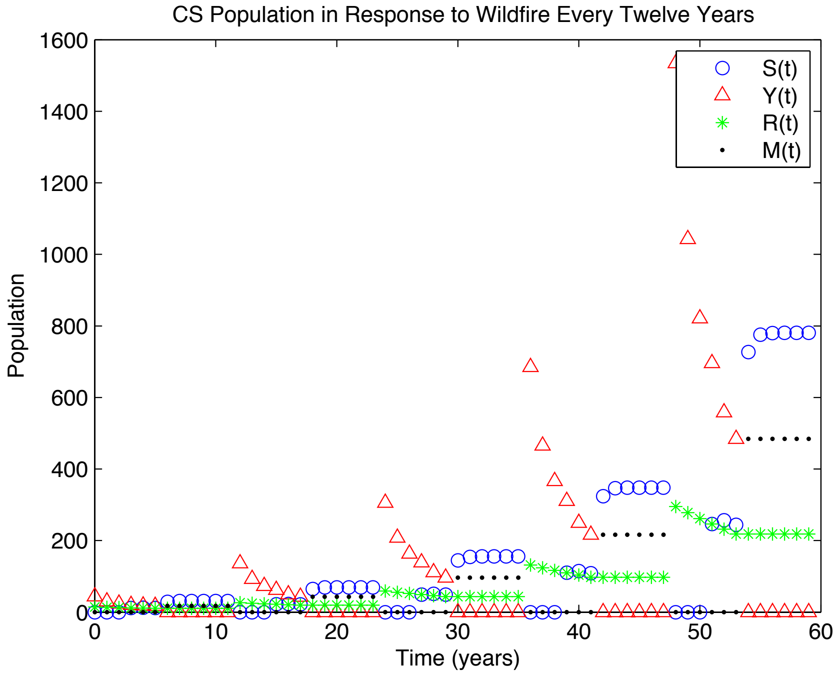

Figure 4.7: C.S. Typical Fire Frequency, High Germination Rate

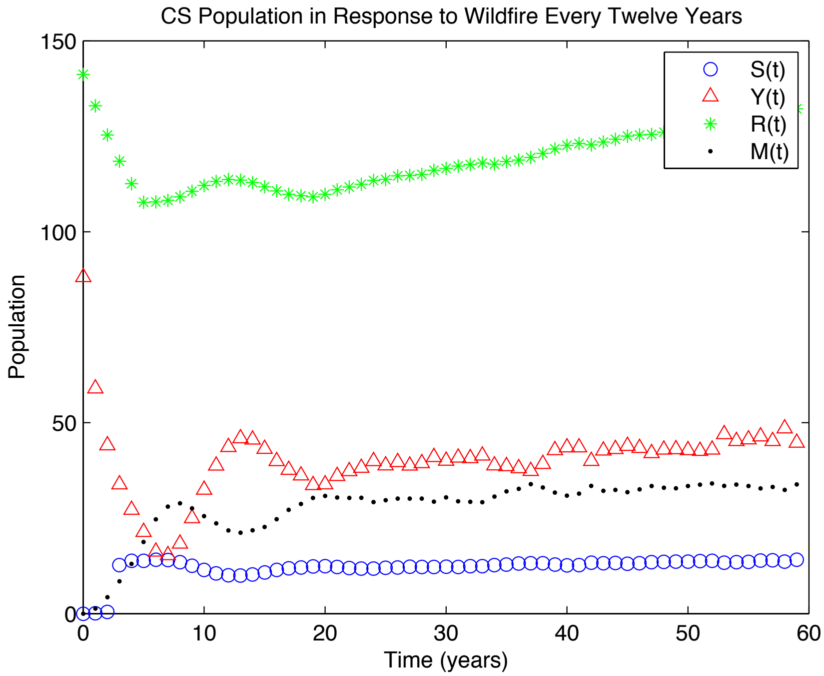

Figure 4.8: C.S. Average of 1000 Simulations with Poisson Distributed Fire and Maturation

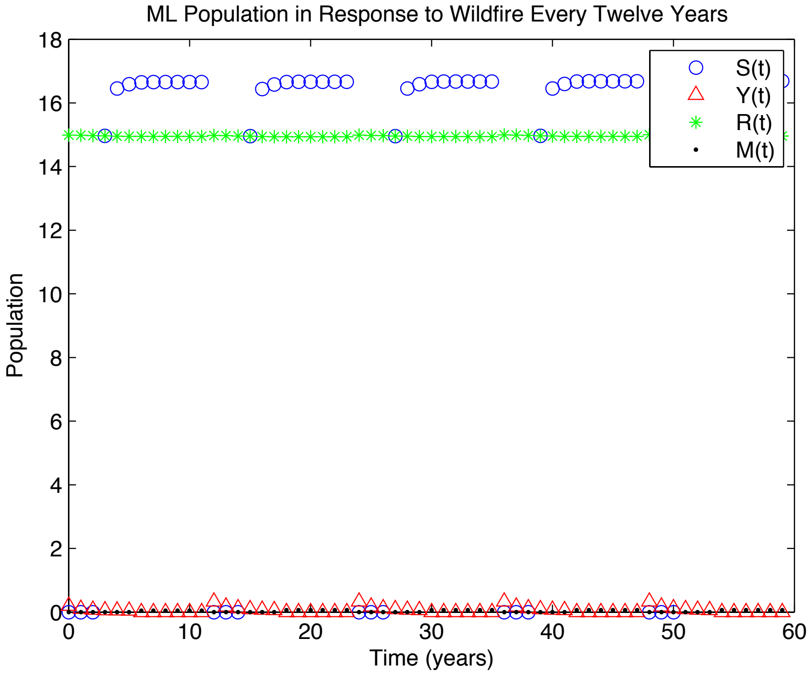

Figure 4.9: M.L. Typical Fire Frequency, Typical Germination Rate

the major elements affecting the two carrying capacities are the two fractions k_Y and k_R. To match the results of the NS model, we assume the product of ϕ, k_Y and k_R is constant. We begin by setting one of the two fractions constant while the other one is varied. Assuming k_R is constant, the log-log plot of ϕ vs. k_Y is linear with slope -0.97 as shown in Figure 4.10.

$$\ln\phi = -\ln k_Y - 1.8 \tag{19}$$

Solving (19) for the product of ϕ and k_Y, a constant of $e^{-1.8}$ is equal to that product. We attempted to find a similar relationship between ϕ and k_R to that between ϕ and k_Y. However, simulations were run many times without getting the result close to the guess. In the future, we will continue to search for a relationship between ϕ and k_R.

5 Spatial Model

This section gives an overview of the assumptions used in the spatial model. The conditions stated in the previous population models are still assumed. In the spatial model, individual plant growth is modeled as well interactions with other chaparral species across the landscape. Individual plants have heritable traits that influence both their life history and the growth of their post-fire generation. When a large, well-mixed community of plants is present on the Santa Monica Mountains, natural events

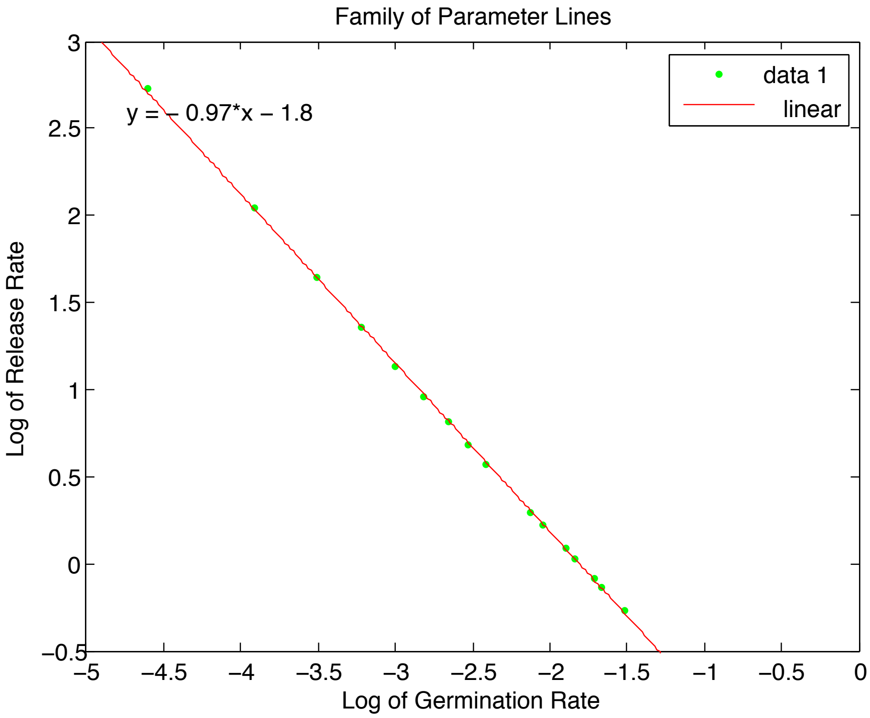

Figure 4.10: Log-log plot of ϕ versus k_Y

such as growth, competition between plants, seed dispersal, germination, and death are updated annually. As mentioned in the population model in post-fire times, we observe different growth behavior for seedlings and resprouts of various chaparral species. Resprouts (*Malosma laurina, Rhus ovata, and Ceanothus spinosus*) grow larger on average than seedlings of NS plant types (e.g. *Ceanothus megacarpus*) [11]. This is also reflected in the data collected and in the fitted growth curves of the four species of chaparral stated above [2].

5.1 Seed Dispersal Assumptions

Assumptions regarding parameters such as germination rates, predation rates, seedling and resprout survival rates are based on communications with Davis [2]. We assume seed dispersal to be exponentially distributed with a mean dispersal distance from the parent plant. Furthermore, seeds are assumed to encircle the parent plant according to a uniformly distributed angle.

5.2 Life history parameters & Growth curves

Our study site is located behind the School of Law at Pepperdine University. The site is designed as a grid on which plants are associated to poles, and each pole is spaced

10 meters from its neighbor poles. One-hundred plants are tagged and each plant is assigned to a permanently indexed pole.

We assume that plant size grows exponentially. Using data on certain plant attributes such as height, crown diameter, and basal diameter we fitted average growth curves using nonlinear regression. These average growth curves are then used to model individual plant growth (See Figures 5.1 and 5.2). The generic form of growth curves is,

$$a(1-\exp(-bx) \tag{20}$$

where the growth limit, a, and the growth rate, b, vary for different species. Assuming that these parameter values are normally distributed, we obtain a 95% confidence interval for mean growth rate and mean growth limit. Table 5.1 with the growth parameters for (20) is presented below. With knowledge of the growth patterns for the four chaparral species we can select a set of growth parameters for an individual plant from the appropriate distribution.

Table 5.1: Growth Function Parameters

Height (cm)

Species	Growth limit (95% C.I)	Growth rate (95% C.I)
Ceanothus spinosus (Cs)*	200.9 (91.2, 223)	.586 (.296,.876)
Malosma laurina (Ml)*	282.8 (245.5, 320.2)	.5857 (.467,.705)
Rhus ovata (Ro)*	90.08 (72.15, 108)	.779 (.2313,1.327)
Ceanothus megacarpus (Cm)†	179.7 (-15.65, 375)	.1054 (-.04208,.2529)

Crown Diameter (cm)

Species	Growth limit (95% C.I)	Growth rate (95% C.I)
Ceanothus spinosus (Cs)*	157.1 (91.2,223)	.5401 (-.1229,1.203)
Malosma laurina (Ml)*	246.6 (202.6,290.7)	.4818 (.2311,.7324)
Rhus ovata (Ro)*	162.2 (116.2,208.7)	.5264 (.1114,.9414)
Ceanothus megacarpus (Cm)†	97.51 (-176.7,371.8)	.0877 (-.217,.3926)

Basal Diameter

Species	Growth limit (95% C.I)	Growth rate (95% C.I)
Ceanothus spinosus (Cs) *	47.43 (26.72,68.13)	.332 (-.0093,.674)
Malosma laurina (Ml)*	68.25 (61.41,75.09)	.9583 (.0524,1.393)
Rhus ovata (Ro)*	85.94 (-1.698,173.6)	.2135 (-.2174,.6443)
Ceanothus megacarpus (Cm)†	1.58 (.2233,2.937)	.1634 (-.04529,.372)

* Facultative Sprouters † Nonsprouter

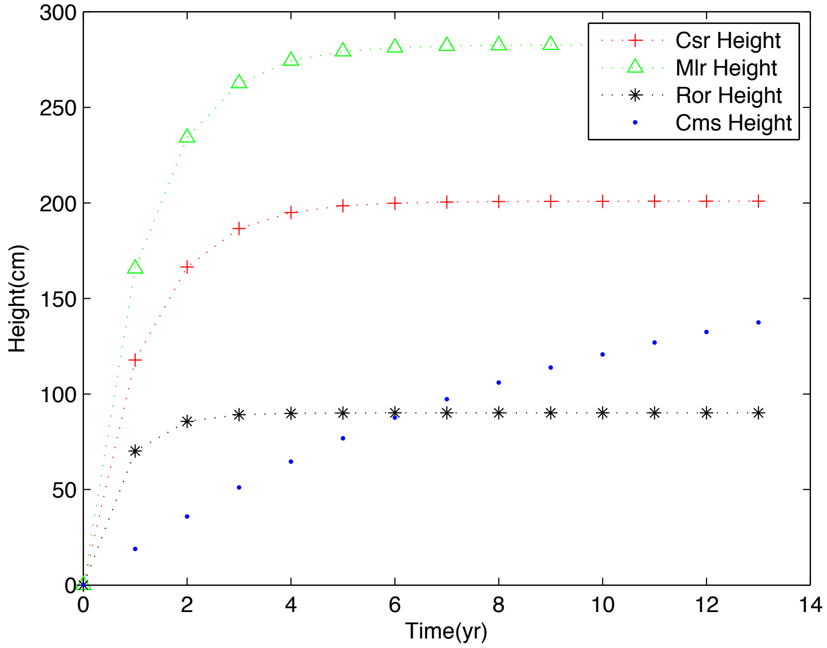

Figure 5.1: Average Heights of Chaparral

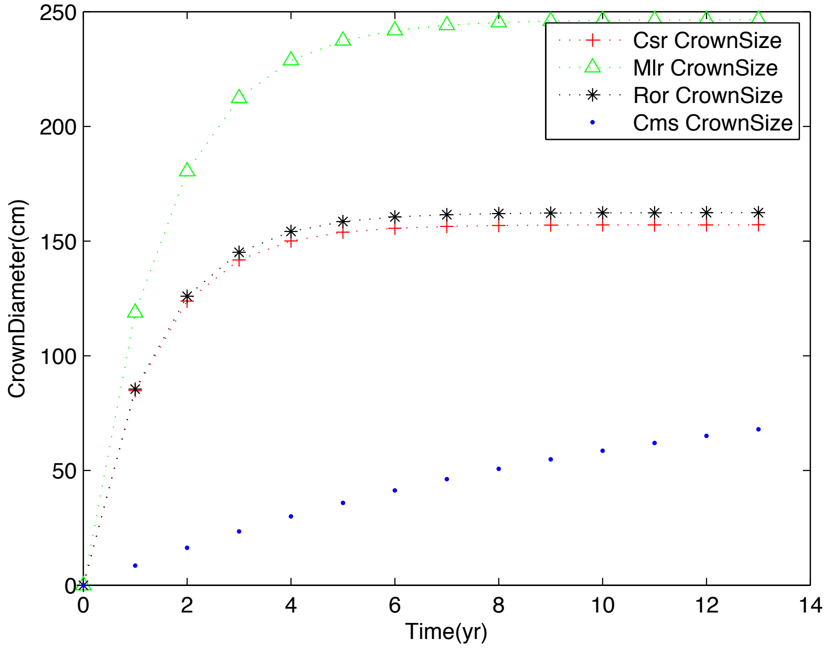

Figure 5.2: Average Crowns of Chaparral

6 Program design

6.1 Program modeling Approaches - Class in Matlab

The spatial model is based around individual plants that grow, drop seeds and sprout from seeds and/or resprout from the burnt base post fire. Thus we construct classes in matlab to model the spatial dynamic of four chaparral species. Some of the key classes include a plant class, plant grid class and seedbank class.

The plant class stores plant attributes such as age, species, location and size. Unique growth curve parameters assigned to each new plant are drawn from a gaussian distribution with given average and standard deviation. It also calls for survival rates for seedlings and resprouts of different species of chaparral, which are stored in a plant type class. The plant class also contains member functions that help plants grow and determine collisions between plants as well as reset the parameters for resprouts and generate plant death which follows a fractional survivalship (logistic decay of plant population).

Next, the plant grid class stores the plants generated in plant class to an indexed pole on a 2D grid given dimension and distance between poles, so that the functions that act on plants are carried out based on pole location. This mirrors the point quarter data collection system used at the study site. Additionally, the plant grid class helps to plot plants and reduce simulation running time as it checks crowding and shading for groups of plants by comparing them only at neighboring poles. That is, it does not compare every single plant on the grid to every other plant on the grid. The seedbank class disperses seeds from the plants, and predates the seeds prior to germination. After fire hits, the seedbank class then takes the germinated seeds and turns them into new plants to be stored in the plant grid. All non-germinated seeds are then removed from the grid. Once the plants reach their maturation years, their seeds are again added to the seedbank class and a fraction of them show up as new plants post fire completing the generation cycle.

6.2 Spatial Simulation

In order to resemble a realistic landscape on SMM, we run spatial simulation that start out with 50 randomly generated plants 10 years dated from the last fire. (Figure 6.1). When fire hits, all plants get wiped out and newly immerged younglings and resprouts show up as small black dots on the grid.(Figure 6.2) With annual update, the resprouts and younglings go through death and growth. (Figure 6.3) Some existing plant disappears from the grid and others grow following their particular growth curve behavior (Figure 6.4 and 6.5) and if zoomed into some of the patches of plants, we see collisions are making deforming circular shapes to deform representing a realistic crowding situation. (Figure 6.6) The model currently runs very basic simulations.

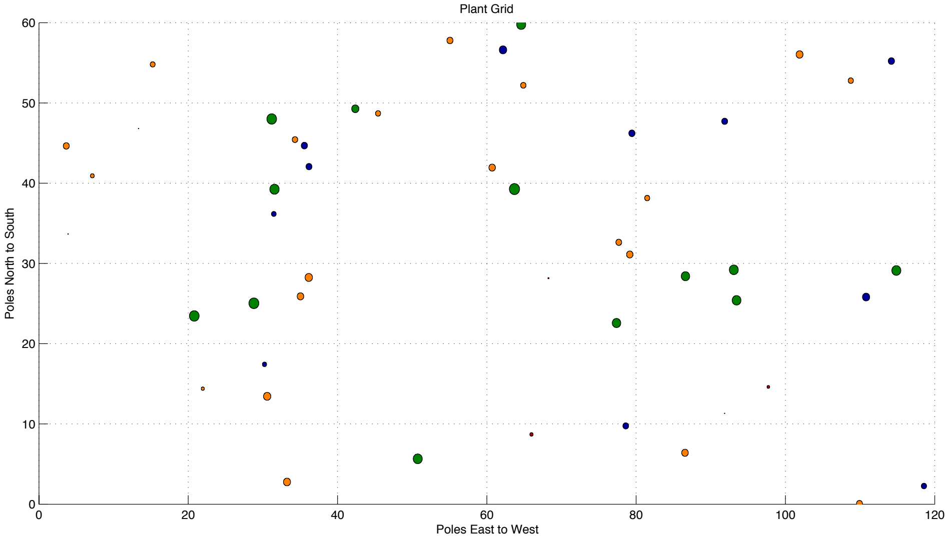

Figure 6.1: Initial Distribution of 50 Plants

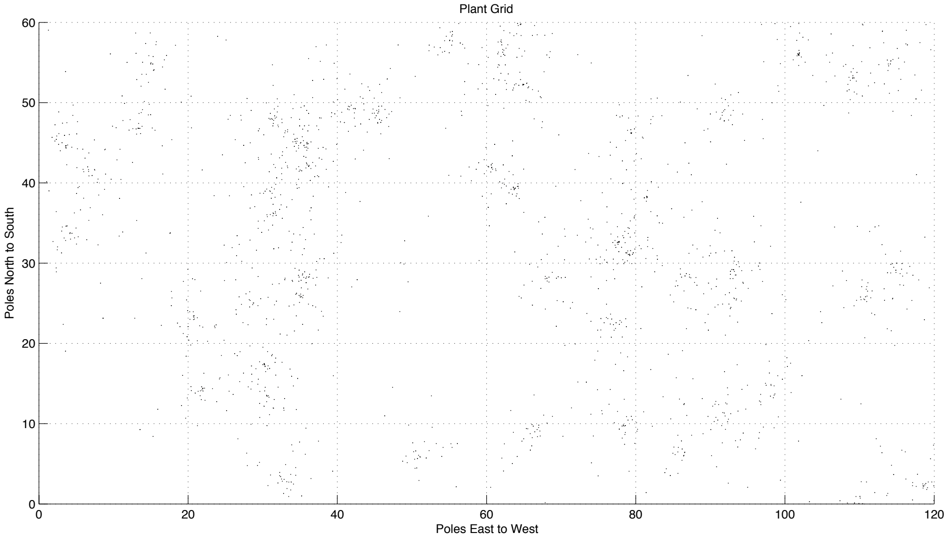

Figure 6.2: Seedlings and Resprouts Post Fire

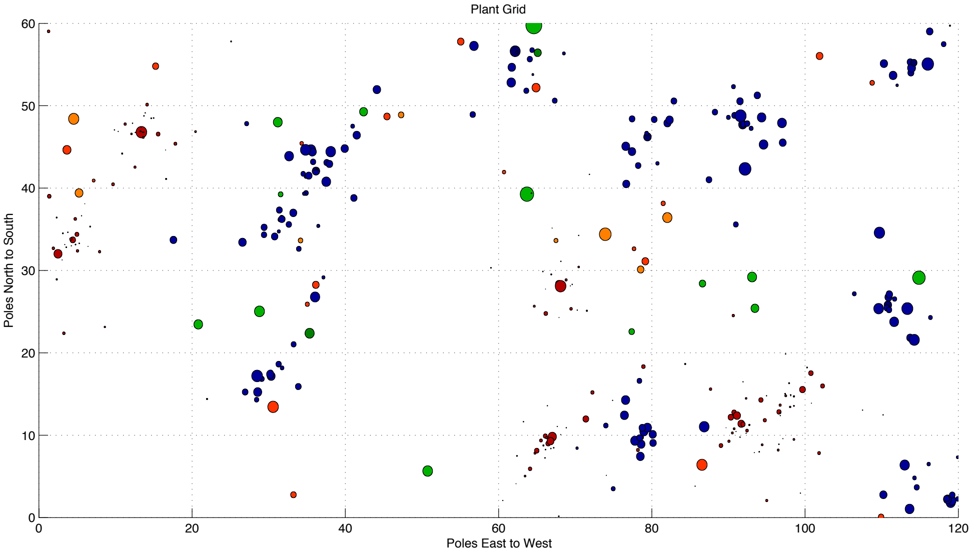

Figure 6.3: Plants One Year Post Fire

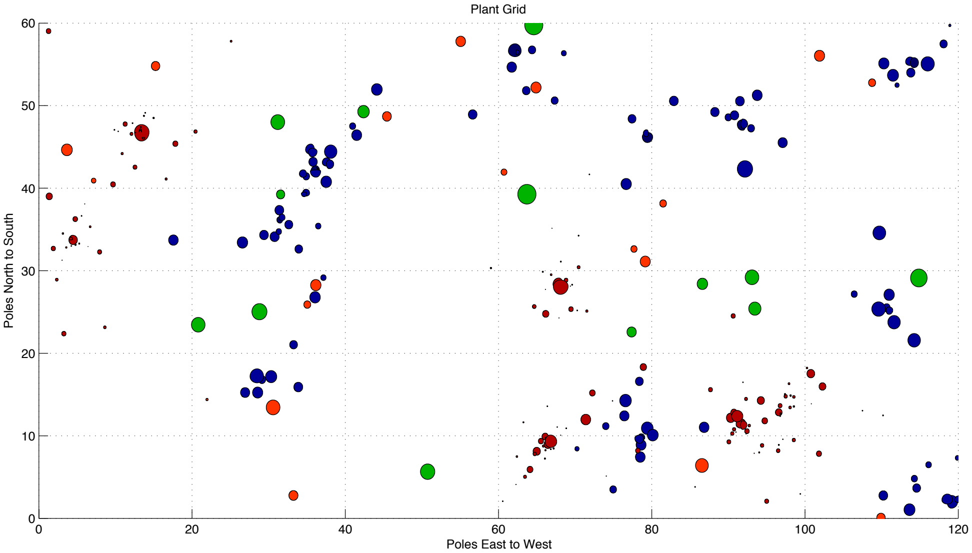

Figure 6.4: Plants Two Years Post Fire

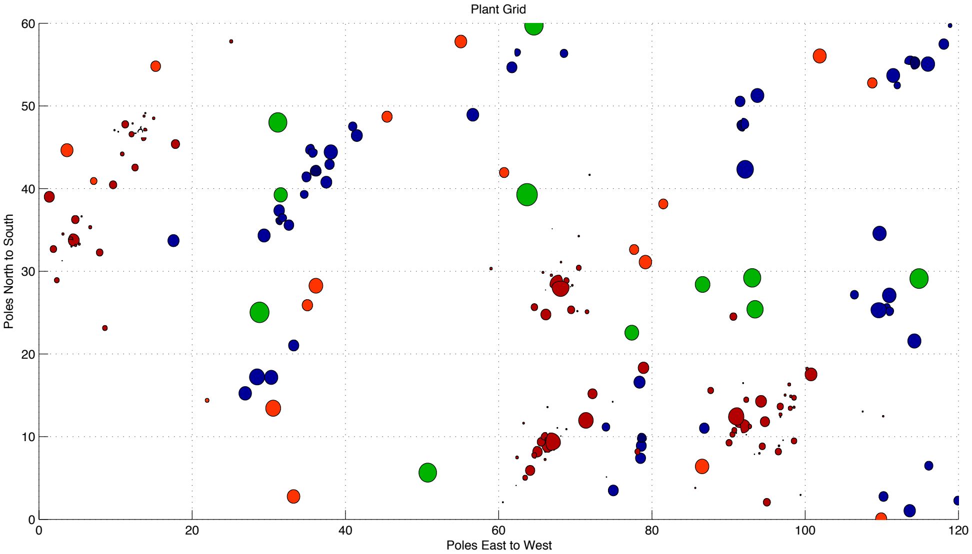

Figure 6.5: Plants Four Years Post Fire

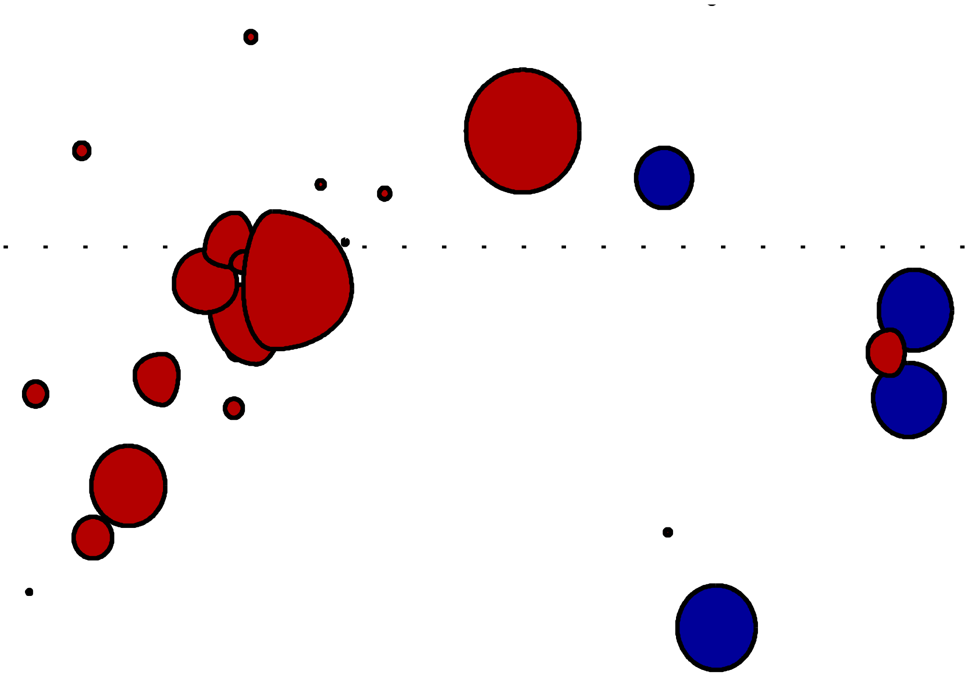

Figure 6.6: Zoom In Plants Crowding

7 Future Work

The models we have designed are not exhaustive, and in order to better reflect nature further parameters must be considered including but not limited to: competition between species for resources, animal herbivory limiting plant growth, water stress tolerance, epidemics, and natural plant thinning. These factors are currently absent from our temporal models, but the spatial model does account for competition by impeding plant growth if two organisms are positioned side-by-side. However, this impedence is unbiased in its selection when in reality certain plant species negatively affect the growth of others–we noticed in our data gathering that *Malosma laurina* growing nearby *Ceanothus spinosus* negatively impacted the latter plant. Furthermore, we have thus far considered germination rates, seed predation rates, and seed release rates to be fixed; we hope to model these parameters as random processes having variance, particularly under certain environmental conditions such as drought or heavy precipitation. Moreover, it is important that we complete a parameter study for FS and OS plant types analogous to that of NS plant types to be understand when equilibriums may arise. Lastly, much of our spatial simulations require an exorbitant amount of time to complete as the code is extensive and has not be optimized for performance. Designing better algorithms to execute the simulation will expedite the production of results.

References

[1] Jack Ainsworth and Troy Alan Doss, *Natural history of fire & flood cyles*, California Coastal Commission, August 1995, Prepared as a presentation to the Post-Fire Hazard Assessment Planning and Mitigation Workshop at the University of California, Santa Barbara.

[2] S. Davis, personal communication, 2011.

[3] Danielle Le Fer and V. Thomas Parker, *The effect of seasonality of burn on seed germination in chaparral: The role of soil moisture*, MADRON O **52** (2005), 166–174.

[4] Jon Keeley, *Seed-germination patterns in fire-prone mediterranean-climate regions*, Ecology and Biogeography of Mediterranean Ecosystems in Chile, California, and Australia. (1995), 239–273.

[5] Jon E. Keeley, *Seed production, seed populations in soil, and seedling production after fire for two congeneric pairs of sprouting and nonsprouting chaparal shrubs*, Ecology **58** (1977), no. 4, pp. 820–829 (English).

[6] R. B. Pratt, A. L. Jacobsen, R. Mohla, F. W. Ewers, and S. D. Davis, *Linkage between water stress tolerance and life history in seedlings of nine chaparral species (rhamnaceae)*, Journal of Ecology **96** (2008), 1252–1265, Both discusses the water stress tolerance of our considered plant populations, and further outlines basic assumptions of the non-sprouting, facultative sprouting, and obligate sprouting plant types.

[7] Klaus W. H. Radtke, *Living more safely in the chaparral-urban interface*, Tech. report, United States Department of Agriculture, 1983.

[8] Daisuke Satoh and Shigeru Yamada, *Parameter estimation of discrete logistic curve models for software reliability assessment*, Japan Journal of Industrial and Applied Mathematics **19** (2002), 39–53, Presents several discrete analogs to logistic curves. Used in the construction of seedling mortality, regression lines are presented in order to estimate parameters that aide in fitting seedling death rate curves based upon empirical data.

[9] William H. Schlesinger, John T. Gray, David S. Gill, and Bruce E. Mahall, *Ceanothus megacarpus chaparral: A synthesis of ecosystem processes during development and annual growth*, Botanical Review **48** (1982), no. 1, pp. 71–117 (English).

[10] Dylan Schwilk, *Plant evolution in fire-prone environments*, Ph.D. thesis, Stanford University, 2002.

[11] C.M. Thomas and S. Davis, *Recovery patterns of three chaparral shrub species after wildfire*, Oecologia **80** (1989), no. 3, 309–320.

[12] Taylor R.S. Witter, M. and S. Davis, *Vegetation response to wildfire and fire histroy in the santa monica mountains, california*, Flora and Ecology of the Santa Monica Mountains (2007), 173–194.

[13] C. Zammit and P. H. Zedler, *Organisation of the soil seed bank in mixed chaparral*, Vegetatio **111** (1994), no. 1, pp. 1–16 (English).

[14] Charles A. Zammit and Paul H. Zedler, *Size structure and seed production in even-aged populations of ceanothus greggii in mixed chaparral*, Journal of Ecology **81** (1993), no. 3, pp. 499–511 (English).