Problem 1 Center of mass in two dimensions.

(a) Find the mass and center of mass of a triangular lamina with vertices $(0,0),(2,0)$, and $(0,1)$ if the density function is $\delta(x,y)=1+x+3y$.

(b) Find the mass and center of mass of a semicircular lamina $x^2+y^2\leq R^2, x\geq 0$ if the density function is $\delta(x,y)=C\sqrt{x^2+y^2}$ for some constant C.

Problem 2 **Center of mass in three dimensions.** Suppose you are eating an ice cream cone. You determine the ice cream cone can be modeled as a solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. If the density of the ice cream cone has uniform density $\delta(x, y) = 1$, determine the mass and center of mass of the ice cream cone.

Problem 3 Probability. The joint density function for a random variables X and Y is

$$f(x,y) = \begin{cases} C(x+y) & \text{if } 0 \le x \le 4, 0 \le y \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant C.

(b) Find $P(X \le 3, Y \ge 1)$.

(c) Find $P(X + Y \le 1)$.

Problem 4 Change of Variables.

(a) Evaluate $\iint_R (x-3y)dA$ where R is the triangular region with the vertices $(0,0),(2,1),(1,2)$. Use the transformation $x=2u+v, y=u+2v$.

(b) Evaluate $\iint_R x^2 dA$ where R is the region bounded by the ellipse $9x^2+4y^2=36$. Use the transformation $x=2u, y=3v$.