Problem 1 Let $\mathcal{D}$ be the region in $\mathbb{R}^3$ bounded by the surfaces (described in cylindrical coordinates) $\theta = z$, $\theta = z + \pi/2$, $z = 0$, $z = \pi/2$, and $r = 1$.

(a) Sketch the region $\mathcal{D}$.

(b) Find the volume of $\mathcal{D}$.

(c) Compute $\iiint_{\mathcal{D}} xyz \, \mathrm{d}V$.

Problem 2 For each $h \in [0, 1]$, let $\mathcal{R}_h$ be the region in $\mathbb{R}^3$ bounded by the surfaces $z = 0$, $z = h$, and $x^2 + y^2 + z^2 = 1$.

(a) Sketch the regions $\mathcal{R}_1$ and $\mathcal{R}_{\frac{1}{2}}$.

(b) Let $f(h) = \iiint_{\mathcal{R}_h} \mathrm{d}V$. Explain why $f'(h) = \pi(1 - h^2)$.

(c) Find $f(h)$ by evaluating the integral $\iiint_{\mathcal{R}_h} \mathrm{d}V$ using cylindrical coordinates.

(d) Find $f(h)$ by evaluating the integral $\iiint_{\mathcal{R}_h} \mathrm{d}V$ using spherical coordinates.

(e) Evaluate

$$\iiint_{\mathcal{R}_{1/2}} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \, \mathrm{d}V.$$

Problem 3 Let $\mathcal{E}$ be the region defined by $x \geq 0$, $y \leq x$, $z \geq 0$, $x^2 + y^2 \leq 4$, and $xz \leq y$. Evaluate

$$\iiint_{\mathcal{E}} (z - x^2 - y^2) \, \mathrm{d}V.$$

Problem 4 Let $\mathcal{E}$ be the region in the first octant bounded by $x^2 + y^2 + z^2 = 4$.

(a) Evaluate

$$\iiint_{\mathcal{E}} (x - y + 3z) \, \mathrm{d}V.$$

(b) Evaluate

$$\iiint_{\mathcal{E}} (x^2 + y^2 - z^2) \, \mathrm{d}V.$$

(c) Griff makes a piece of Jell-O® whose shape is $\mathcal{E}$. Because Griff isn't very good at cooking, the density of her Jello-O® is very uneven: it is given by the function $\delta(x, y, z) = \frac{x^2+y^2}{x^2+y^2+z^2}$. Find the mass of Griff's Jell-O®.

Problem 5 Let $\mathcal{S}$ be the region bounded by $x = -1$, $x = 1$, $y + z = 0$, $y - z = 0$, and $y^2 + z^2 = 2$. Find the volume of $\mathcal{S}$ (Hint: sketch what S looks like and think about which coordinate system to use).