Problem 1 Ben sits his first midterm for Math 32B and gets the following question:

Let W be the region above $z = x^2 + y^2$ and below $z = 5$ and bounded by $y = 0$ and $y = 1$. Calculate $\int_W y dV$.

(a) After working on it for a while, he eventually arrives at the answer of $-\frac{\sqrt{5}}{5}$. Does this seem reasonable? Why/Why not?

(b) Here is Ben's working out:

$$= \int_{-\sqrt{5}}^{\sqrt{5}} \int_0^1 \int_{x^2+y^2}^{5} y \, dz \, dy \, dx = \int_{-\sqrt{5}}^{\sqrt{5}} \int_0^1 \frac{y^2}{2} \Big|_{x^2+y^2}^{5} dy \, dx$$

$$= \int_{-\sqrt{5}}^{\sqrt{5}} \int_0^1 \frac{25}{2} - \frac{(x^2+y^2)^2}{2} dy \, dx$$

$$= \iint \frac{25}{2} dy dx - \iint \frac{(x^2+y^2)^2}{2} dy dx$$

$$= 25\sqrt{5} - \frac{1}{2} \int x^4 + y^4 dy dx$$

$$= 25\sqrt{5} - \frac{1}{2} \int_{-\sqrt{5}}^{\sqrt{5}} \int_0^1 x^4 dy dx - \frac{1}{2} \int_{-\sqrt{5}}^{\sqrt{5}} \int_0^1 y^4 dy dx$$

$$= 25\sqrt{5} - \frac{1}{2} x^5 \Big|_{-\sqrt{5}}^{\sqrt{5}} - \sqrt{5} \cdot \frac{1}{5} .$$

$$= 25\sqrt{5} - 25\sqrt{5} - \frac{1}{5}\sqrt{5} = -\frac{1}{5}\sqrt{5} .$$

Can you find all the mistakes that Ben made?

(c) Did you find Ben's answer hard or easy to read? Come up with things that Ben did right in his answer that made it easy to read and things that Ben could do to help make his answer easier to read.

(d) Rewrite Ben's answer correctly and compare your answer with other people in your group. Don't just compare the end result but read each other's solutions and see what they did/didn't do that made it easy to read.

Problem 2 We will integrate $\int_{-\infty}^{\infty} e^{-x^2} dx$ in this question using a trick with polar coordinates.

(a) Let $I = \int_{-\infty}^{\infty} e^{-x^2}dx$. Justify why $I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}dxdy$.

(b) Don't question your TA when they say that

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}dxdy = \lim_{R\to\infty}\iint_{B_R} e^{-x^2-y^2}dxdy$$

where B_R is the ball centered at zero of radius R.

(c) Use polar coordinates to calculate $\iint_{B_R} e^{-x^2-y^2}dxdy$.

(d) What is I equal to?

Problem 3 It is always a good idea to keep an eye out for symmetries in double and triple integrals in order to reduce calculations needed. Without calculating anything, what are the following integrals?

(a) $\int_{-1}^{1}\int_{-2}^{2}\int_{0}^{4-x^2} x^2z^3\ln(x^2+y^2-z^2+2)dydxdz$

(b) $\int_{-1}^{1}\int_{-1}^{1} 2+y^2x^3-x^2\sin(y)dxdy$

(c) $\int_{1}^{2}\int_{1}^{2}\frac{x^2}{x^2+y^2}dxdy$

Problem 4 A riddle: I am a two-variable function. I can tell how far a given point is from the origin but not how far from either the x or y-axis. My average value over any annulus is always equal to the reciprocal of the sum of the inner and outer radius. What function am I?

Problem 5 It's time for the annual triple integral beauty pageant. After hearing about the amazing prize (admiration from your TA and fellow students), you decide to enter it. Can you come up with an interesting triple integral that evaluates to exactly 2020?