Problem 1 Double integrals over rectangles. Evaluate the following integrals:

(a) $\iint_R (x+y)dxdy$ where $R = \{(x,y) : 0 \le x \le 3, 0 \le y \le 1\}$

(b) $\int_0^1 \int_1^2 \frac{xe^x}{y} dydx$ (*Hint:* use integration by parts, $\int udv = uv - \int vdu$)

(c) $\int_0^1 \int_0^\pi r\sin^2\theta d\theta dr$ (*Hint:* use $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$)

Problem 2 Double integrals over general regions. Evaluate the following integrals:

(a) $\int_0^2 \int_x^{2x} xy\,dy\,dx$

(b) $\iint_D 2x\sqrt{y^2 - x^2}\,dx\,dy$ where $D = \{(x, y) : 0 \leq y \leq 1, 0 \leq x \leq y\}$

Problem 3 Switching the order of integration.

(a) Sketch the region of integration and change the order of integration:

$$\int_0^4 \int_{x^2}^{16} f(x, y)\,dy\,dx = \int_{\_}^{\_} \int_{\_}^{\_} f(x, y)\,dx\,dy$$

(b) Evaluate the integral by switching the order of integration:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dxdy$$

Problem 4 **Average value.** The average value of a function $f(x, y)$ over a region D is given by

$$f_{ave} = \frac{1}{A} \iint_D f(x, y) dA$$

where $A = \iint_D 1 dA$, i.e. the area of the region D. Use this formula to find the average value of the given function over the given domain.

(a) $f(x, y) = xy^2$, the rectangle with vertices $(-1, 0), (-1, 3), (2, 3), (2, 0)$

(b) $f(x, y) = x - y$, the triangle with vertices $(0, 0), (2, 0), (2, 2)$

Problem 5 Triple Integrals. Evaluate the following integrals:

(a) $\int_0^1 \int_0^1 \int_0^1 (9 - xz^2) dx dy dz$

(b) $\int_0^2 \int_0^z \int_0^y 3ze^{-y^2} dx dy dz$