Problem 1 Use Green's Theorem to find the area of the ellipse $(x/3)^2 + (y/4)^2 = R^2$, where $R > 0$.

Problem 2 Let $\mathcal{S}$ be portion of the ellipsoid $(x/3)^2 + (y/4)^2 + (z/5)^2 = 1$ lying between the planes $z = 0$ and $z = 1$, oriented with outward-pointing normal vectors.

(a) Draw $\mathcal{S}$ with its boundary orientation.

(b) Let F be the vector field $\langle -y, -z, 1\rangle$. Check that F is the curl of $\langle -y, yz, xz\rangle$.

(c) Use Stokes' Theorem to compute $\iint_{\mathcal{S}} F \cdot \mathrm{d}S$.

Problem 3 Let $F = \langle x^3, y^4, z^2\rangle$ and let $\mathcal{S}$ be the portion of the surface $z = x^2 - y^2$ lying inside $x^2 + y^2 = 1$, oriented in the positive z-direction. Use Stokes' Theorem to compute

$$\oint_{\partial\mathcal{S}} F \cdot \mathrm{d}r.$$

Problem 4 Let $f(x, y, z)$ be an infinitely differentiable function of 3 variables. Let $\mathcal{S}$ be the level surface $f(x, y, z) = 0$.

(a) Explain why the flux of ∇f through $\mathcal{S}$ must be nonzero, assuming that $\mathcal{S}$ has nonzero surface area.

(b) Use this to show that the vector field $\langle x, y, z\rangle$ does not have a vector potential.

Problem 5 Let $\mathcal{S}$ be the surface given in spherical coordinates by $\rho = 1 + \sin(\phi)$, oriented outward.

(a) Sketch the surface $\mathcal{S}$.

(b) Use Stokes' Theorem to compute $\iint_{\mathcal{S}} F \cdot \mathrm{d}S$, where $F = \langle -4z^3, -2x, -3y^2\rangle$.