STUDENT NAME: ______________________________

STUDENT ID NUMBER: ______________________________

DISCUSSION SECTION NUMBER: ______________________________

Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work—your work must both justify and clearly identify your final answer. No books, notes or calculators are allowed. You must simplify results of function evaluations when it is possible to do so.

For instructor use only

Page	Points	Score
2	10	
3	10	
4	16	
5	10	
6	16	
7	14	
8	10	
9	10	
10	12	
11	12	
Total:	120	

1. [10 pts] Integrate the function $f(x, y, z) = x$ over the solid region in the first octant ($x \geq 0, y \geq 0, z \geq 0$) bounded above by $z = 8 - 2x^2 - y^2$ and below by $z = y^2$.

2. [10 pts] Use the transformation $x = 2u + v$, $y = u + 2v$ to evaluate $\int\int_R (x - 3y)dA$ where R is the triangular region with vertices $(0, 0), (2, 1), (1, 2)$ in the xy-plane. Be sure to draw both the original region R AND the resulting region in the uv-plane.

3. [8 pts] Find the volume enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the plane $x + 2y + z = 5$, and bounded below by the plane $x + y + z = -1$.

4. [8 pts] Prove that div curl $\overrightarrow{\mathbf{F}}(x, y, z) = 0$ for any vector field $\overrightarrow{\mathbf{F}}$.

5. Suppose $f(x, y, z)$ is a scalar function, and $\overrightarrow{\mathbf{F}}(x, y, z)$ is a vector field (these two may be totally unrelated). Suppose we have a closed curve $\mathcal{C}$ in $\mathbb{R}^3$ and we know that $\oint_{\mathcal{C}} f ds = 5$ and $\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}} = 6$.

   (a) [3 pts] What is the value of $\oint_{-\mathcal{C}} f ds$? No justification needed.

   (b) [3 pts] What is the value of $\oint_{-\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}}$? No justification needed.

   (c) [4 pts] Is it possible that $\overrightarrow{\mathbf{F}} = \overrightarrow{\boldsymbol{\nabla}} f$? Explain.

6. [6 pts] Let C be the unit circle in $\mathbb{R}^2$, oriented counter-clockwise. Find $\oint_C \langle 2x - y, 2y + x\rangle \cdot d\overrightarrow{\mathbf{r}}$.

7. Let $f(x, y) = x^2 + 3y$, and let L be the line segment from $(3, 0)$ to $(0, 4)$.

   (a) [5 pts] Find $\int_L f(x, y) ds$.

   (b) [5 pts] Find $\int_L \overrightarrow{\boldsymbol{\nabla}} f(x, y) \cdot d\overrightarrow{\mathbf{r}}$

8. Let $\mathcal{S}$ be the cone given by equation $\phi = \frac{\pi}{6}$ in spherical coordinates.

   (a) [4 pts] Find a parametrization $G(u, v)$ for $\mathcal{S}$.

   (b) [4 pts] Find the angle between the tangent vectors $\overrightarrow{\mathbf{T}}_u$ and $\overrightarrow{\mathbf{T}}_v$ at the point $(1, 0, \sqrt{3})$ (given in (x, y, z) coordinates).

   (c) [6 pts] Find an equation for the tangent plane to the surface at that same point.

9. [10 pts] The plane $x + 2y + 2z = 3$ has a finite portion $\mathcal{S}$ lying in the first octant $(x, y, z \geq 0)$. Suppose that this plane carries a charge density of $\delta_Q(x, y, z) = e^z$. Find the total charge Q on $\mathcal{S}$. Be sure to have a good sketch of your parametrization domain $\mathcal{D}$ if necessary.

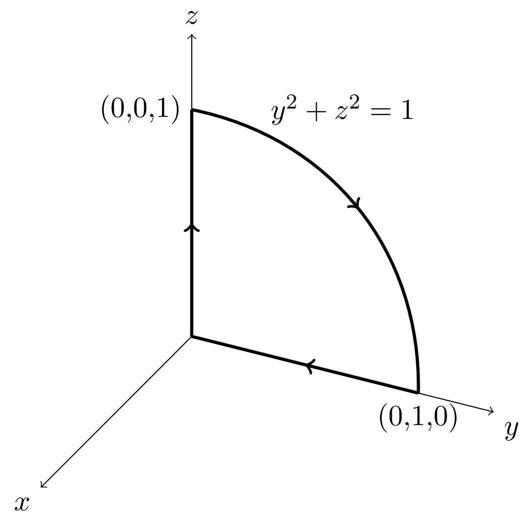
10. [10 pts] Compute the surface integral (flux) $\iint_{\mathcal{S}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{S}}$ of $\overrightarrow{\mathbf{F}} = \langle y, x, e^{xz} \rangle$ over the surface $\mathcal{S}$ defined by the equations $x^2 + y^2 = 9, x \geq 0, y \geq 0, -3 \leq z \leq 3$, with outward-pointing normal. Be sure to have a good sketch of your parametrization domain $\mathcal{D}$ if necessary.

11. [6 pts] Prove that, if $\overrightarrow{\mathbf{F}}$ is a conservative vector field, then the flux of $\text{Curl}(\overrightarrow{\mathbf{F}})$ through any surface $\mathcal{S}$ (closed or not closed) must equal zero.

12. [6 pts] Find the volume of a region $\mathcal{W}$ if we know that

$$\iint_{\partial\mathcal{W}} \left\langle x + xy + z \,,\, x + 3y - \frac{1}{2}y^2 \,,\, 4z \right\rangle \cdot d\overrightarrow{\mathbf{S}} = 16$$

13. [6 pts] Evaluate $\oint_{\mathcal{C}} \langle y, z, x \rangle \cdot d\overrightarrow{\mathbf{r}}$ where $\mathcal{C}$ is the closed curve illustrated below (HINT: Stokes' Theorem).



14. TRUE/FALSE (circle your answer, no justification needed)

(a) [3 pts] Suppose that $\text{Curl}\,\overrightarrow{\mathbf{F}}(x, y, z) = \overrightarrow{\mathbf{0}}$ throughout some simply connected domain. Then there is one and only one potential function $f(x, y, z)$ such that $\overrightarrow{\mathbf{F}} = \overrightarrow{\boldsymbol{\nabla}} f$.

TRUE *FALSE*

(b) [3 pts] If $\overrightarrow{\mathbf{F}}(x, y, z)$ is conservative on some domain (simply connected or not), then we must have $\text{Curl}\,\overrightarrow{\mathbf{F}} = \overrightarrow{\mathbf{0}}$.

TRUE *FALSE*

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