1. Let $\mathbf{F}(x, y) = \langle y^2 + 1, 2xy - 2\rangle$. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot dr$ where $\mathcal{C}$ is:
   (a) The line segment from $(0, 0)$ to $(1, 1)$.
   (b) The path from $(0, 0)$ to $(1, 1)$ that first moves in a straight line to $(0, 1)$ and then moves in a straight line to $(1, 1)$.
   (c) Reconcile your answers with the fundamental theorem of conservative vector fields.

2. Let $f(x, y) = \sin x + x^2 y$ and let $\mathbf{F} = \nabla f$. Let $\mathcal{C}$ be the part of the parabola $y = x^2$ going from $(0, 0)$ to (π, π^2).
   (a) Compute $\int_{\mathcal{C}} \mathbf{F} \cdot dr$ using the definition of vector line integrals.
   (b) Compute $\int_{\mathcal{C}} \mathbf{F} \cdot dr$ using the fundamental theorem of conservative vector fields.

3. Consider the vector field $\mathbf{F} = \langle 2xy^2z^2 + e^{x^2}, 2x^2yz^2 - e^{y^2}, 2x^2y^2z\rangle$. Let $\mathcal{C}$ be the part of the curve $\mathbf{r}(t) = \langle t, t^2, t\rangle$ $0 \leq t \leq 1$. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

   Before you embark on doing this problem, discuss with your group different ways that you might approach this problem.

4. A vector field $\mathbf{F}$ has curl zero and is defined on all of $\mathbb{R}^2$ except for $(0, 0)$ and $(2, 0)$.
   (a) Show that if $\mathcal{C}$ is the circle of radius R with $R > 1$ centered at $(1, 0)$ then $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ is independent of R.
   (b) Integrating $\mathbf{F}$ over a small circle centered at $(0, 0)$ oriented counterclockwise gives 2 and integrating $\mathbf{F}$ over a small circle centered at $(2, 0)$ oriented clockwise gives -1.

   What is the integral of $\mathbf{F}$ over the circle of radius R with $R > 1$ centered at $(1, 0)$ oriented counterclockwise?