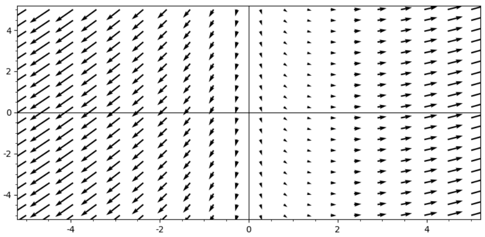
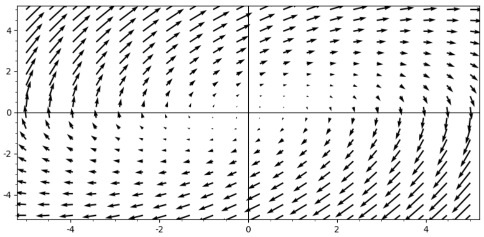
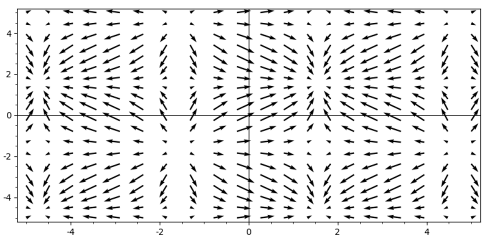


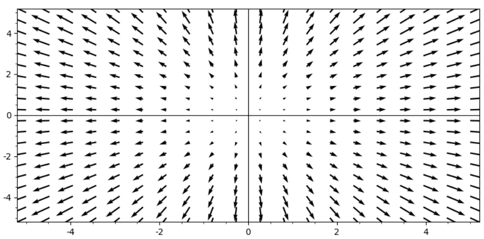
(a)



(b)



(c)



(d)

Figure 1

1. Match the vector field with its plot in the above figure. Talk with your group and relate characteristics of the plots to the functions.
   (a) $\mathbf{F}(x, y) = \langle x, y \rangle$
   (b) $\mathbf{F}(x, y) = \langle y, y - x \rangle$
   (c) $\mathbf{F}(x, y) = \langle \cos x, \cos y \rangle$
   (d) $\mathbf{F}(x, y) = \langle x, x - 2 \rangle$

2. Which of the above vector fields can you conclude are *not* conservative? For the others, can you find a potential function?

3. Show that if $\mathbf{F}(x, y, z)$ is a vector field with smooth component functions then the divergence of the curl of $\mathbf{F}$ is 0, i.e. $\nabla \cdot (\nabla \times F) = 0$. Give an explanation for why this is not surprising in light of what you know about $\cdot$ and $\times$ from 32a.

4. Try to determine whether or not the vector field $\mathbf{F}(x, y, z) = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$ is conservative. If you think it is conservative, find a potential function.

5. Consider the vector field $\mathbf{F}(x, y) = \langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \rangle$.
   (a) Compute $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathcal{C}$ is the circle of radius r centered at the origin, oriented counterclockwise. This computation shows that $\mathbf{F}$ is *not* conservative. (We'll talk about this more in class on Wednesday)
   (b) Show that the curl of $\mathbf{F}$ is 0. Is this surprising in light of numbers 2 and 4 on this worksheet and that fact that $\mathbf{F}$ is not conservative (and what we've talked about in class, if you are doing this worksheet later in the week)? Give a resolution to the conundrum. **Hint:** What are the domains of all these vector fields?