1. (a) Find a change of coordinates map G that takes the unit square $[0,1]\times[0,1]$ to the parallelogram with vertices $(0,0),(2,1),(1,2)(3,3)$.

   (b) Find the Jacobian of G.

   (c) Find a change of coordinates map G' that takes the unit square $[0,1]\times[0,1]$ to the parallelogram with vertices $(2,1),(4,2),(3,3),(5,4)$.

   (d) Find the Jacobian of G' and give a geometric explanation for the similarity between the Jacobian of G and that of G'.

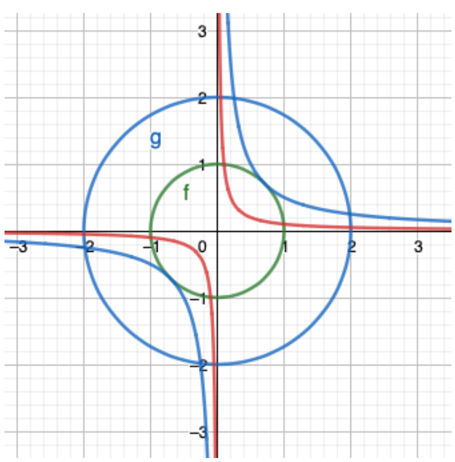
2. Consider the region $\mathcal{D}$ defined by $1 \le x^2 - y^2 \le 4$ and $0 \le y \le \frac{3x}{5}$. In this problem you'll set up an integral to compute $\iint_{\mathcal{D}} e^{x^2-y^2}\,\mathrm{d}A$.

   Consider the change of coordinates $G(u,v) = \left(\frac{v}{2}+\frac{u}{2v}, \frac{v}{2}-\frac{u}{2v}\right)$. Recall from class that the inverse of this coordinate change is given by $G^{-1}(x,y) = (x^2-y^2, x+y)$

   (a) Find a region $\mathcal{R}$ of the uv-plane so that $G:\mathcal{R}\to\mathcal{D}$ is a change of coordinates map (so G is onto and one-to-one on the interior of $\mathcal{R}$). **Hint:** Start by finding 4 curves in the uv-plane that map to the 4 curves forming the boundary of $\mathcal{D}$.

   (b) Give an iterated integral in uv-coordinatees to compute $\iint_D e^{x^2-y^2}\,\mathrm{d}A$ (No need to compute the actual integral, but it is an integral you can compute).

3. Consider the region of the part of the first quadrant $\mathcal{D}$ defined by $1 \le x^2+y^2 \le 4$ and $1/10 \le xy \le 1/2$ and $y \ge x$. There is a change of coordinates G that takes the rectangle $[1,4]\times[1/10,1/2]$ in the uv-plane to $\mathcal{D}$, and the *inverse* of this change of coordinates is given by $G^{-1}(x,y) = (x^2+y^2, xy)$.



(a) What is the absolute value of the Jacobian of G^{-1}? (It should be a function of x and y). Pay attention to signs!

(b) Compute $\iint_{\mathcal{D}} y^2 - x^2 \, \mathrm{d}A$

(c) Bonus problem: Note that the system of inequalites $1 \leq x^2 + y^2 \leq 4$ and $1/10 \leq xy \leq 1/2$ defines *four* different regions of the plane. Each of these regions can be described by a change of coordinates G that takes the rectangle $[1, 4] \times [1/10, 1/2]$ in the uv-plane to the region where again *inverse* of this change of coordinates is given by $G^{-1}(x, y) = (x^2 + y^2, xy)$, but for each of these regions G itself has a different formula. Find all 4 formula for G and say which of these four regions goes with which formula.