1. We showed that the circle of radius a has the concise description in polar coordinates as $r = a$.
   Find a description of the circles $(x-a)^2 + y^2 = a^2$ and $x^2 + (y-a)^2 = a^2$ in polar coordinates. (What if a is negative?)

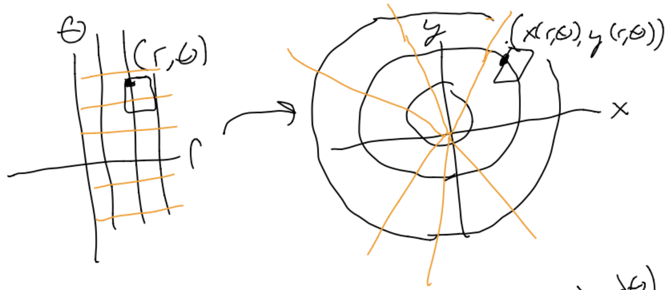
2. Compute the area inside the curves $r = \cos\theta$ and $r = \sin\theta$.

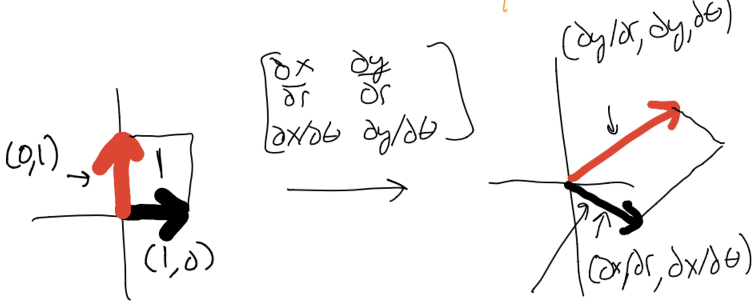
3. (a) Plot the curve $r = 1 + cos(5\theta)$ (it should look something like a flower).
   (b) Find the area of one "petal" of the curve $r = 1 + \cos 5\theta$. You will probably need to use the double angle formula.

4. If a solid in a region $\mathcal{W} \subset \mathbb{R}^3$ has density given by $\partial(x, y, z)$ then its mass is given by $\iiint_{\mathcal{W}} \partial(x, y, z)\ \mathrm{d}V$.
   Suppose that we are measuring in meters and consider a solid that is above the plane $z = 0$, below the paraboloid $z = 4 - (x^2 + y^2)$, and outside the cylinder $x^2 + y^2 = 1$. Suppose that the density of this solid is inversely proportional to the distance from the z-axis and that the density of this solid along the boundary where the paraboloid hits the xy-plane is $1/2\ kg/m^3$.
   Compute the mass of this solid.

5. In this problem you will find the area of the ellipse $(x/a)^2 + (y/b)^2 = 1$. We'll use a distorted version of polar coordinates. We'll measure points in the plane by the angle θ the line from the origin to the point makes with the x-axis and the value of r for which the point lies on the ellipse $\left(\frac{x}{ar}\right)^2 + \left(\frac{y}{br}\right)^2 = 1$.
   (a) Using these coordinates what point in the xy-plane does the value (r, θ) correspond to?
   (b) Describe the ellipse $(x/a)^2 + (y/b)^2 = 1$ in these new "distorted polar coordinates".
   (c) What is the distortion factor for area with these coordinates?
   For this, consider the map $(r, \theta) \mapsto (x(r, \theta), y(r, \theta))$ from the first part of the question. Differentiating this gives the matrix $\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{bmatrix}$, and the area distortion factor at (r, θ) is the determinant of this

matrix, the quantity $\dfrac{\partial x}{\partial r}\dfrac{\partial y}{\partial \theta} - \dfrac{\partial y}{\partial r}\dfrac{\partial x}{\partial \theta}$. See the picture below.





the area of the square
$[0,1]\times[0,1]$ under the this map is
$\partial x/\partial r \cdot \partial y/\partial \theta - \dfrac{\partial y}{\partial r} \cdot \dfrac{\partial x}{\partial \theta}$

(d) What is the area enclosed by this ellipse?