Remember Stoke's theorem: if $\mathcal{S}$ is an oriented surface and $\partial\mathcal{S}$ has the boundary orientation then if $\mathbf{F}$ is a vector field with continuous partial derivative then $\iint_{\mathcal{S}} \nabla \times \mathbf{F} \cdot dS = \oint_{\partial\mathcal{S}} \mathbf{F} \cdot dr$.

1. Let $\mathbf{F}$ be the vector field $\langle x, y, xyz\rangle$ and let $\mathcal{S}$ be the part of the plane $2x + y + z = 2$ that lies in the first octant oriented upwards. Verify that Stoke's theorem holds in this example by explicity computing $\iint_{\mathcal{S}} \nabla \times F \cdot dS$ and $\oint_{\partial\mathcal{S}} F \cdot dr$.

2. Let $\mathcal{S}_1$ be the surface $x^2+y^2+4z^2 = 4$ where $z \geq 0$ and let $\mathcal{S}_2$ be the surface $z = 4-x^2-y^2$ where $z \geq 0$, where each surface is oriented with the normal pointed upwards. If $\mathbf{F}$ is a vector field with continuous partial derivatives explain why $\iint_{\mathcal{S}_1} \nabla \times F \cdot dS = \iint_{\mathcal{S}_2} \nabla \times F \cdot dS$.

3. (a) Let $\mathcal{D}$ be the disc $x^2 + y^2 \leq 4$ with upward pointing orientation and let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle xz\sin(yz), \cos(yz), e^{x^2+y^2}\rangle$. What is $\iint_{\mathcal{D}} \mathbf{F} \cdot dS$?

   (b) Let S be the part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$ with downward pointing orientation. What is $\iint_{S} \mathbf{F} \cdot dS$ (here $\mathbf{F}$ is the vector field from the previous part of the problem)? **Hint:** Does $\mathbf{F}$ have a vector potential (don't try to explicity find what it is!)?.

4. Let $\mathcal{W}$ be the part of the solid cylinder $x^2 + y^2 \leq 1$ where $0 \leq z \leq 1$, let $\partial\mathcal{W}$ be the boundary of this solid with the outwards pointing orientation, and let $\mathbf{F} = \langle xy, yz, xz\rangle$.

   (a) Directly compute $\iint_{\partial\mathcal{W}} \mathbf{F} \cdot dS$.

   (b) Directly compute $\iiint_{\mathcal{W}} \operatorname{div} \mathbf{F}\, dV$.

   (c) Compare your answers– what do you notice?