Midterm 1 practice 2

UCLA: Math 32B, Fall 2019

Instructor: Noah White
Date:

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: Solutions

ID number:

Question	Points	Score
1	9	
2	11	
3	10	
4	10	
Total:	40	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				
(i)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) If $\mathcal{R} = [-2, 0] \times [-1, 2]$, the integral $\iint_{\mathcal{R}} 2 \,\mathrm{d}A$ is equal to

   A. 12

   B. 10

   C. 15

   D. 14

   (b) (1 point) If $\mathcal{R} = [0, 2] \times [0, 1]$, the integral $\iint_{\mathcal{R}} 3xy^2 \,\mathrm{d}A$ is equal to

   A. -2

   B. 1

   C. -1

   D. 2

   (c) (1 point) If $\mathcal{B} = [-1, 0] \times [12, 13] \times [3, 4]$, the integral $\iiint_{\mathcal{B}} -1 \,\mathrm{d}V$ is equal to

   A. -1

   B. 1

   C. -2

   D. 2

(d) (1 point) If $\mathcal{R} = [1,2] \times [-2,2] \times [3,6]$, the integral $\iiint_{\mathcal{B}} y e^{\ln(x^2+z^2)}\,\mathrm{d}A$ is equal to

A. 0

B. 2

C. -1

D. $3\pi^2$

(e) (1 point) If $\mathcal{B} = [0,\pi/2] \times [0,\pi] \times [0,1]$, the integral $\iiint_{\mathcal{B}} z\cos x + z\sin y\,\mathrm{d}V$ is equal to

A. $-\pi$

B. 0

C. $\pi/2$

D. π

Hint: integrate in the order $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

(f) (1 point) The volume of the region bounded by $z = 1 - x^2 - y^2$ in the first octant, i.e. where $x, y, z \geq 0$, is equal to

A. π

B. $\pi/8$

C. $\pi/4$

D. 0

(g) (1 point) If $\mathcal{D}$ is the region bounded by the y-axis, $y = x$ and $x^2 + y^2 = 4$, where $x, y \geq 0$ then after changing to polar coordinates, the integral $\iint_{\mathcal{D}} x^2 + y^2 \, \mathrm{d}A$ becomes

A. $\int_0^{\pi/4} \int_0^4 r^2\theta \, \mathrm{d}r \, \mathrm{d}\theta$

B. $\int_0^{\pi/2} \int_0^4 r^3 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta$

C. $\int_{\pi/4}^{\pi/2} \int_0^2 r^2 \, \mathrm{d}r \, \mathrm{d}\theta$

D. $\int_{\pi/4}^{\pi/2} \int_0^2 r^3 \, \mathrm{d}r \, \mathrm{d}\theta$

(h) (1 point) The integral of $e^{x^2+y^2}$ over the disc $x^2 + y^2 \leq 1$ is

A. π

B. $2\pi(e-1)$

C. $\pi(e-1)$

D. $e-1$

(i) (1 point) If $\mathcal{D}$ is the region between the curves $y = x^3$ and $y = \sqrt[3]{x}$ in the first quadrant then $\mathcal{D}$ has the description

A. $0 \leq x \leq 3, \quad \sqrt[3]{x} \leq y \leq x^3$

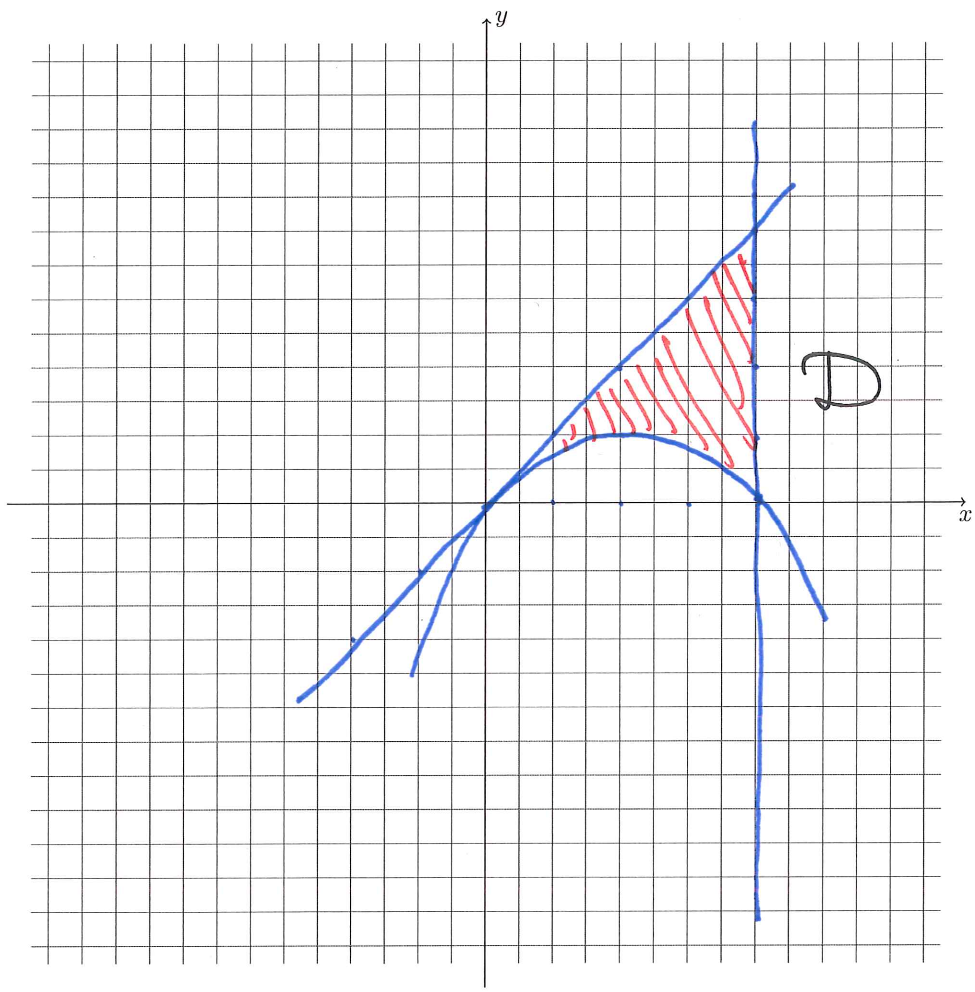
B. $0 \leq x \leq 3, \quad x^3 \leq y \leq \sqrt[3]{x}$

C. $0 \leq x \leq 1, \quad \sqrt[3]{x} \leq y \leq x^3$

D. $0 \leq x \leq 1, \quad x^3 \leq y \leq \sqrt[3]{x}$

2. In this question we will consider the region $\mathcal{D}$ which bounded by the lines
   - $y = x$,
   - $y = x(1-x)$, and
   - $x = 1$.

   (a) (2 points) Sketch the region $\mathcal{D}$ on the graph provided.



(b) (1 point) Is $\mathcal{D}$ vertically simple region, horizontally simple, both or neither?. *Hint: verically simple means $f(x) \le y \le g(x)$ and horizontally simple means $f(y) \le x \le g(y)$.*

Solution: vertically.

(c) (2 points) Express $\mathcal{D}$ either as a horizontally simple region or as vertically simple.

Solution:

$0 \le x \le 1 \qquad x(1-x) \le y \le x$

(d) (2 points) Write the integral

$$\iint_{\mathcal{D}} x \, dA$$

as an iterated integral.

Solution:

(e) (4 points) Evaluate the integral in the previous part.

Solution: 1/4

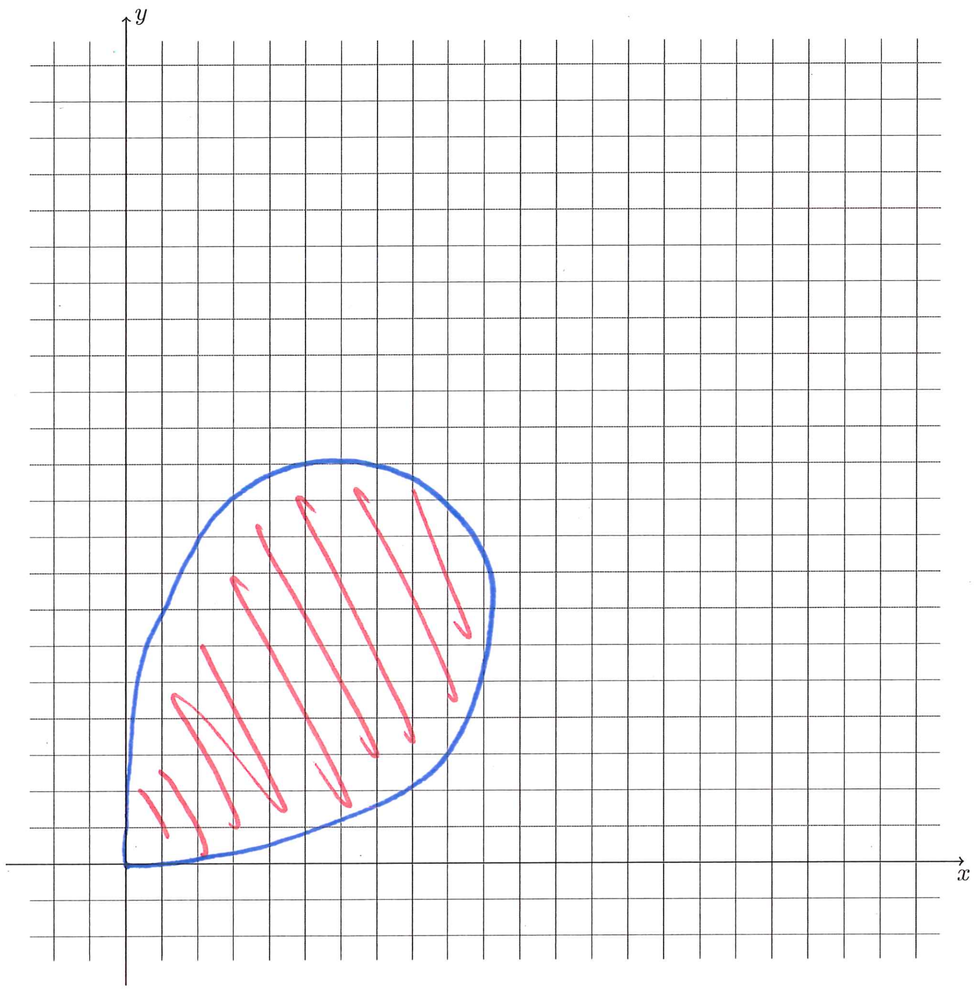
d) $$\iint_D x\,dA = \int_0^1 \int_{x(1-x)}^{x} x \, dy \, dx$$

e) $$= \int_0^1 x^2 - x^2(1-x) \, dx$$

$$= \int_0^1 x^3 \, dx = \frac{1}{4} x^4 \Big|_0^1 = \frac{1}{4}$$

3. In this question we will consider the region $\mathcal{D}$ which bounded by curve $r = \sin 2\theta$, in the first quadrant.

(a) (2 points) Sketch the region $\mathcal{D}$ on the graph provided (your sketch can be rough, it does not need to be perfect, it just need to show the main features).



(b) (2 points) Write the region enclosed by the curve as a radially simple region, i.e in the form $\varphi \leq \theta \leq \psi$ and $r_1(\theta) \leq r \leq r_2(\theta)$ for some functions r_1 and r_2.

Solution:

$$0 \leq \theta \leq \frac{\pi}{2}$$

$$0 \leq r \leq \sin 2\theta$$

(c) (2 points) Write the area of $\mathcal{D}$ as an iterated integral.

Solution:

(d) (4 points) Calculate the area of $\mathcal{D}$. You may use the fact that $\int \sin^2\theta \, d\theta = \frac{1}{2}(\theta - \sin\theta\cos\theta)$.

Solution:

c) $$\iint_D 1 \, dA_{xy} = \iint_{\mathcal{D}} r \, dA_{r\theta}$$

$$= \int_0^{\pi/2} \int_0^{\sin 2\theta} r \, dr \, d\theta$$

d) $$= \int_0^{\pi/2} \frac{1}{2}\sin^2 2\theta \, d\theta$$

$$= \int_{u=0}^{u=\pi} \frac{1}{4}\sin^2 u \, du$$

$u = 2\theta$

$du = 2\,d\theta$

$$= \frac{1}{8}(u - \sin u \cos u)\Big|_0^{\pi}$$

$$= \frac{\pi}{8}$$

4. Consider the region $\mathcal{E}$ that is bounded by the surface $z = 1 - x^2$ and the planes $z = y$ and $y = 0$.

(a) (4 points) Describe the region in the form

$$\mathcal{E} = \{\ (x, y, z) \in \mathbb{R}^3 \mid (x, y) \in \mathcal{D},\ z_1(x, y) \leq z \leq z_2(x, y)\}$$

for $\mathcal{D}$ a region in the xy-plane. Your answer should specify what $\mathcal{D}$ is.

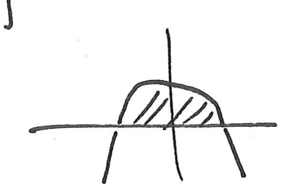
Solution:

(b) (6 points) Compute the volume of the region $\mathcal{E}$.

Solution:

8/15

a) $\mathcal{E} = \{ (x\,y\,z) \mid (x\,y) \in D \quad y \leq z \leq 1 - x^2 \}$



where

D is the region in $\mathbb{R}^2$ bounded by $y = 0$ $\quad y = 1 - x^2$

b) $$\iiint_{\mathcal{E}} 1\,dV = \iint_D \int_y^{1-x^2} dz\, dA_{xy}$$

$$= \int_{-1}^{1} \int_0^{1-x^2} \int_y^{1-x^2} dz\,dy\,dx$$

$$= \int_{-1}^{1} \int_0^{1-x^2} 1 - x^2 - y \; dy\, dx$$

$$= \int_{-1}^{1} \frac{1}{2}(1 - x^2)^2\, dx$$

$$= \int_{-1}^{1} \frac{1}{2}(1 - 2x^2 + x^4)\, dx$$

$$= \frac{1}{2}\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1} = 1 - \frac{2}{3} + \frac{1}{5} = \frac{8}{15}$$