Midterm 1 practice 1

UCLA: Math 32B, Fall 2019

Instructor: Noah White
Date:
Version: practice

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: Solutions

ID number: ____________________

Discussion section: ____________________

Question	Points	Score
1	9	
2	10	
3	12	
4	9	
Total:	40	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				
(i)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (1 point) If $\mathcal{R} = [0,3] \times [-1,4]$, the integral $\iint_{\mathcal{R}} 3 \, \mathrm{d}A$ is equal to

A. 12

B. 0

C. 15

D. 45

(b) (1 point) If $\mathcal{R} = [0,1] \times [-1,1]$, the integral $\iint_{\mathcal{R}} y^3 \sin(x^2 y^2) \, \mathrm{d}A$ is equal to

A. π

B. 0

C. -1

D. $3\pi^2$

(c) (1 point) If $\mathcal{R} = [0,3] \times [2,4]$, the integral $\iint_{\mathcal{R}} x - y \, \mathrm{d}A$ is equal to

A. -9

B. 4.5

C. -4.5

D. 1

(d) (1 point) If $\mathcal{B} = [0,1] \times [0,1] \times [-1,1]$, the integral $\iiint_{\mathcal{B}} -1 \, dV$ is equal to

A. -4

B. 1

C. -2

D. 2

(e) (1 point) If $\mathcal{B} = [0,1] \times [-1,0] \times [0,4]$, the integral $\iiint_{\mathcal{B}} ye^{y^2} \, dV$ is equal to

A. $e - 1$

B. $e^2 - 1$

C. $1 - e$

D. $2 - 2e$

(f) (1 point) If $\mathcal{E}$ is the region bounded by the curves $y = 2x^2$ and $y = 1-x^2$, then $\mathcal{E}$ has the description

A. $-\sqrt{3} \leq x \leq \sqrt{3}, \;\; 2x^2 \leq y \leq 1 - x^2$

B. $-1/\sqrt{3} \leq x \leq 1/\sqrt{3}, \;\; 2x^2 \leq y \leq 1 - x^2$

C. $-1/\sqrt{3} \leq x \leq 1/\sqrt{3}, \;\; 0 \leq y \leq 1$

D. $-\sqrt{3} \leq x \leq \sqrt{3}, \;\; 1 - x^2 \leq y \leq 2x^2$

(g) (1 point) If $\mathcal{D}$ is the disc $x^2 + y^2 \leq 4$, then after changing to polar coordinates, the integral $\iint_{\mathcal{D}} xy \, dA$ becomes

A. $\int_0^{\pi} \int_0^1 r \, dr \, d\theta$

B. $\int_0^{2\pi} \int_0^2 r^2 \sin 2\theta \, dr \, d\theta$

C. $\int_0^{\pi} \int_0^2 r^3 \sin 2\theta \, dr \, d\theta$

D. $\int_0^{2\pi} \int_0^2 r^3 \sin\theta \cos\theta \, dr \, d\theta$

(h) (1 point) The integral of $x^2 + y^2$ over the annulus $4 \leq x^2 + y^2 \leq 16$ is

A. 6π

B. 2π

C. π

D. 120π

(i) (1 point) If $\mathcal{D}$ is the region between the curves $y = x^2$ and $y = \sin(\frac{1}{2}\pi x)$ in the first quadrant then $\mathcal{D}$ has the description

A. $0 \leq x \leq \pi, \ \sin(\frac{1}{2}\pi x) \leq y \leq x^2$

B. $0 \leq x \leq 1, \ \sin(\frac{1}{2}\pi x) \leq y \leq x^2$

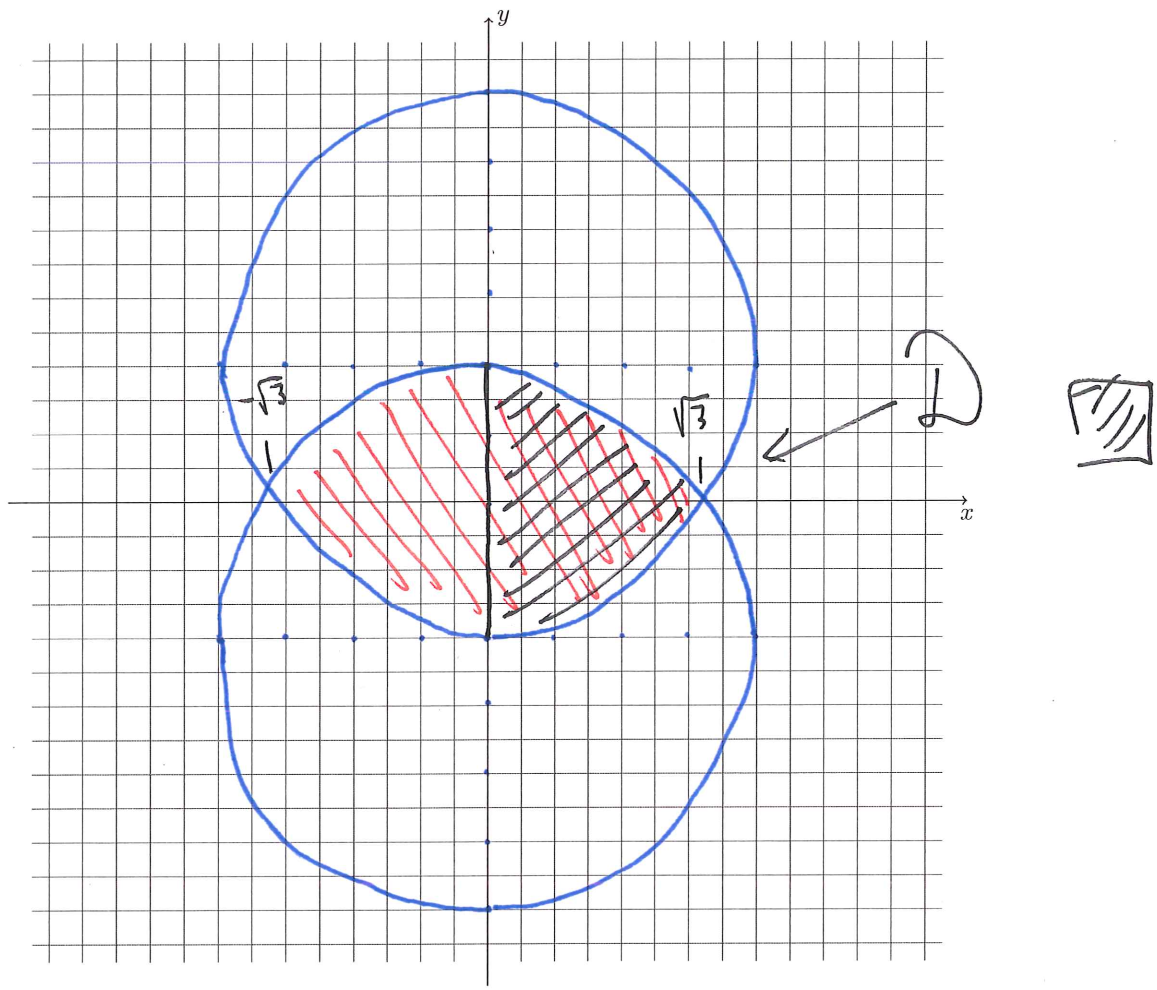
C. $0 \leq x \leq \pi, \ 0 \leq y \leq \sin(\frac{1}{2}\pi x)$

D. $0 \leq x \leq 1, \ x^2 \leq y \leq \sin(\frac{1}{2}\pi x)$

2. In this question we will consider the region $\mathcal{D}$ which is the intersection of

- $x^2 + (y-1)^2 \leq 4$,
- $x^2 + (y+1)^2 \leq 4$, and
- $x \geq 0$.

(a) (2 points) Sketch the region $\mathcal{D}$ on the graph provided.



(b) (2 points) Express $\mathcal{D}$ as a vertically simple region, i.e. in the form $a \leq x \leq b$, $g_1(x) \leq y \leq g_2(x)$.

Solution: $0 \leq x \leq \sqrt{3}$, and $1 - \sqrt{4-x^2} \leq y \leq -1 + \sqrt{4-x^2}$

(c) (2 points) Write the integral

$$\iint_{\mathcal{D}} x \, dA$$

as an iterated integral

Solution:

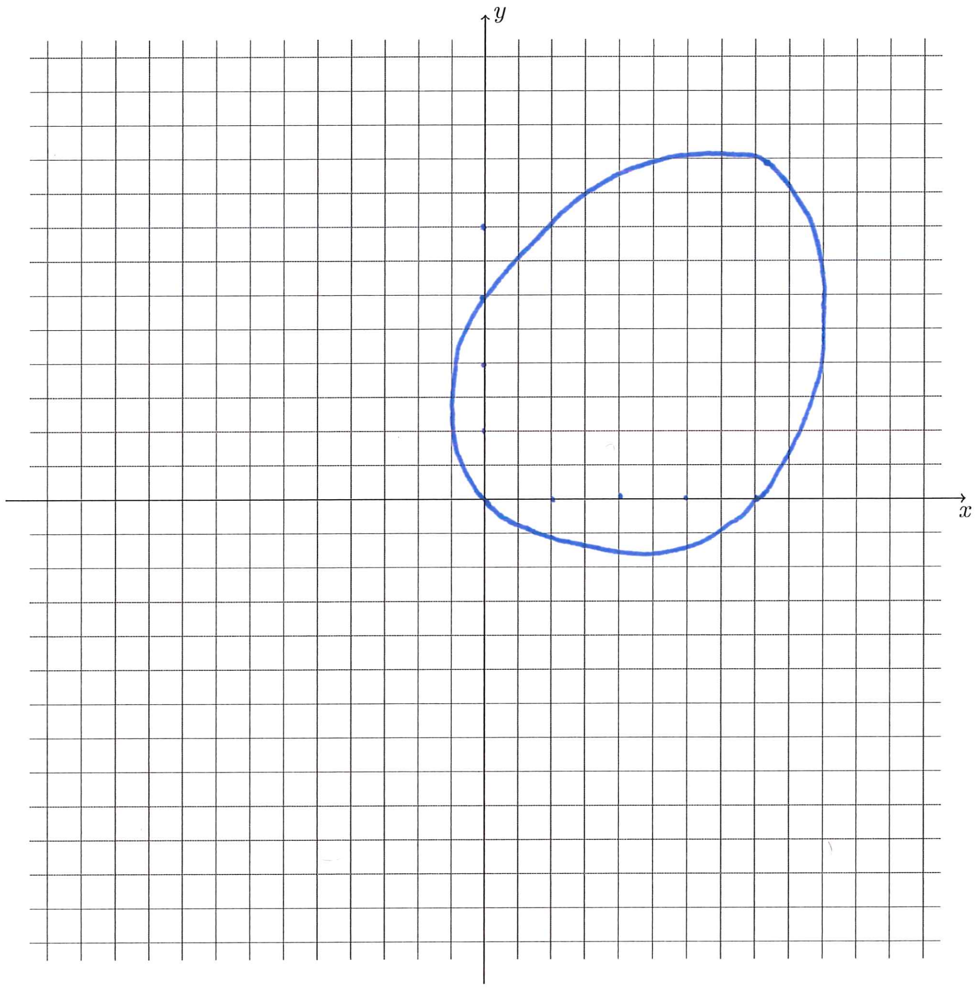
$$\int_0^{\sqrt{3}} \int_{1-\sqrt{4-x^2}}^{-1+\sqrt{4-x^2}} x \, dy \, dx$$

(d) (4 points) Evaluate the integral in the previous part.

Solution: $\frac{5}{3}$

3. In this question, consider the equation $r = \cos\theta + \sin\theta$.

   (a) (4 points) Sketch the curve described by the above equation on the graph provided.



(b) (4 points) Find the area bounded by this curve in the first quadrant.

Solution:

$$\frac{\pi}{4} + \frac{1}{2}$$

(c) (4 points) Integrate the function $f(x, y) = \frac{1}{\sqrt{x^2+y^2}}$ over the *entire* region (not just the first quadrant).

Solution: $2\sqrt{2}$. Note that the limits on your integral should be $-\pi/4$ to $3\pi/4$, not 0 to 2π. This is a little tricky. See Campuswire for explanation

b) $0 \leq \theta \leq \pi/2$

$0 \leq r \leq \sin\theta + \cos\theta$

$$\iint_D 1 \, dA_{xy} = \iint_D r \, dA_{r\theta} = \int_0^{\pi/2} \int_0^{\sin\theta + \cos\theta} r \, dr \, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} (\sin\theta + \cos\theta)^2 \, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} (1 + 2\sin\theta\cos\theta) \, d\theta$$

$$= \frac{\pi}{4} + \int_0^{\pi/2} \sin\theta\cos\theta \, d\theta$$

$$= \frac{\pi}{4} + \cancel{\int} \frac{1}{2}\sin^2\theta \Big|_0^{\pi/2}$$

$$= \frac{\pi}{4} + \frac{1}{2}$$

4. Consider the region $\mathcal{E}$ above the plane $z = 4 - 2y$ and below the paraboloid $z = 4 - x^2 - y^2$.

(a) (4 points) Describe the region in the form

$$\mathcal{E} = \{\ (x, y, z) \in \mathbb{R}^3 \mid (x, y) \in \mathcal{D},\ z_1(x, y) \le z \le z_2(x, y)\}$$

for $\mathcal{D}$ a region in the xy-plane. Your answer should specify what $\mathcal{D}$ is.

Solution: $\mathcal{E}$ is the region given by $(x, y) \in \mathcal{D}$ and $4 - 2y \le z \le 4 - x^2 - y^2$ where $\mathcal{D}$ is the disk centred at $(0, 1)$ with radius 1.

(b) (5 points) Compute the volume of the region $\mathcal{E}$. *Hint: you might find the identity* $\int_0^\pi \sin^4\theta \, d\theta = 3\pi/8$ *useful*

Solution:

$$\frac{\pi}{2}$$

a) note that the intersection of the surfaces are the points

$$(x, y, 4-2y)$$

where $4 - 2y = 4 - x^2 - y^2$

ie $\underline{x^2 + (y-1)^2 = 1}$

hence disk radius 1 at $(0,1)$.

b) The integral ~~sou~~ should ~~by~~ be

$$\iiint_{\mathcal{E}} 1\, dV = \iint_{D} \int_{4-2y}^{4-x^2-y^2} dz\, dA_{xy}$$

$$= \iint_{D} \int_{4-2r\sin\theta}^{4-r^2} dz\; r\, dA_{r\theta}$$

$$= \int_0^{\pi} \int_0^{2\sin\theta} \int_{4-2r\sin\theta}^{4-r^2} r\, dz\, dr\, d\theta$$

$\boxed{\text{or}}$

$$= \int_{-1}^{1} \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \int_{4-2y}^{4-x^2-y^2} dz\, dy\, dx$$