# Worksheet 1

(1) Consider the plane
$$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1.$$
Where does this plane intersect the $x$-, $y$-, and $z$-axes? Call the intersection points $P$, $Q$, and $R$, in that order. Find the vectors $v=\overrightarrow{PQ}$ and $w=\overrightarrow{PR}$ and the cross-product $v\times w$. Is $v\times w$ a normal vector for the plane? Does that make sense in this situation?

intersect at $\underbrace{(2,0,0)}_{P}, \underbrace{(0,3,0)}_{Q}, \underbrace{(0,0,4)}_{R}$.

$\vec{v}=(-2,3,0)$, $\vec{w}=(-2,0,4)$.

Yes it's normal as $\vec{v},\vec{w}$ tangent to the plane by construction (and not parallel)

(2) Find all values of $b$ such that the vectors $\langle 4,-2,7\rangle$ and $\langle b^2,b,0\rangle$ are orthogonal.

$\langle 4,-2,7\rangle\cdot\langle b^2,b,0\rangle=0 \iff$ orthogonal.

$4b^2-2b=0$

$2b(2b-1)=0$

so $b=0, 1/2$.

if $b=0$, I guess this is still technically orthogonal..

(3) Parametrize the curve of intersection of the surfaces $z=x^2$ and $x^2+y^2=1$. (It's the outline of a Pringle!)

$r(t)=(\cos t, \sin t, \cos^2 t)$.

(4) Find the unit vector at $P = (0, 0, 1)$ pointing in the direction along which the function

$$f(x, y, z) = xz + e^{-x^2+y}$$

increases most rapidly.

$$\nabla f = \langle z - 2xe^{-x^2+y}, e^{-x^2+y}, x \rangle$$

Hence $\nabla f(0,0,1) = \langle 1, 1, 0 \rangle$.

(5) Is there a function $f$ such that $\nabla f = \langle y^2, x\rangle$?

$$f = \int y^2 dx = y^2 x + c(y)$$

$f_y = 2xy + c'(y) = x$. this is impossible

Alternatively, if such an $f$ existed, as

$$\frac{\partial f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x} \Rightarrow 2y = 1$$ which isn't correct.

(6) Identify the following surfaces in 3D (sketch them, don't worry if you can't remember their names):

1) $z = x^2 + y^2$ 3) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ 5) $x^2 + y^2 + z^2 = 16$

2) $z^2 = x^2 + y^2$ 4) $z = y^2$ 6) $y = 9 - x^2 - z^2$

1) parraboloid 3) elliptic cylinder 5) sphere

2) cone 4) parabella sheel 6) parraboloid.