Week 1 Notes

Information:

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I'll post my discussion notes here. There is also a link where you can let me know if there is anything you want to do in discussion.
The course webpage can be found at:
http://www.math.ucla.edu/~das/32a.2.19s/

There will be weekly quizzes starting next week which we will do at the beginning of each discussion.

Today: Vectors and Geometry.

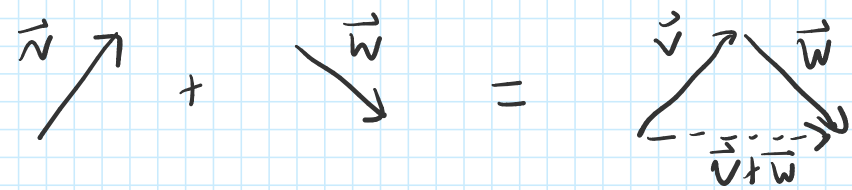
There are two equivalent ways to think about vectors.

1) as a pair or triple of numbers for a vector on the plane or 3-space respectively
2) something that has both direction and length.

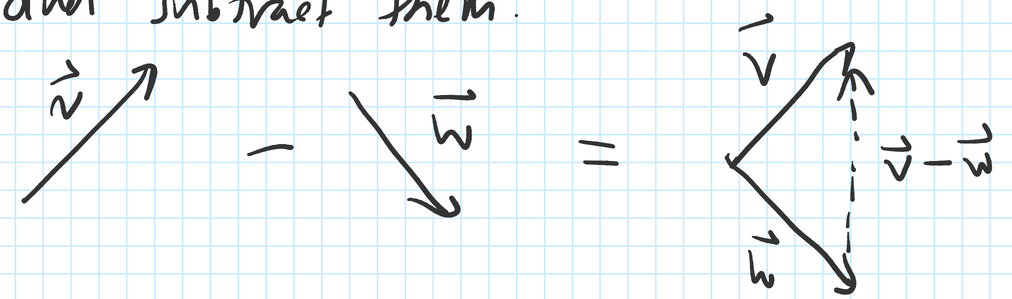
We will focus on the second way today.

Reminders:

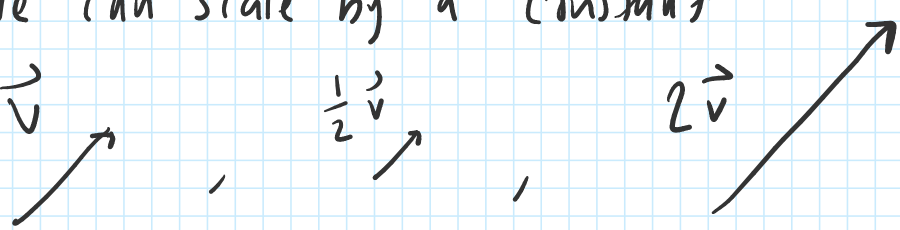
- A vector $\vec{v}$ isn't fixed anywhere, we can move it around as long as we don't change its length or direction.
- Given two vectors $\vec{v}$ and $\vec{w}$, we can add them:



and subtract them:



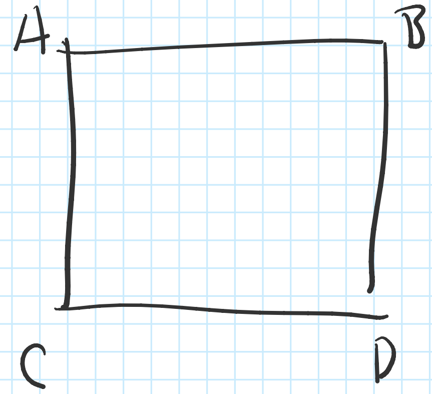
- We can scale by a constant



- $-\vec{v}$ has the same length as $\vec{v}$ but opposite direction

- Given points P, Q. $\overrightarrow{PQ}$ is the vector starting at P and ending at Q.

warning: vectors aren't fixed. So take square ABCD

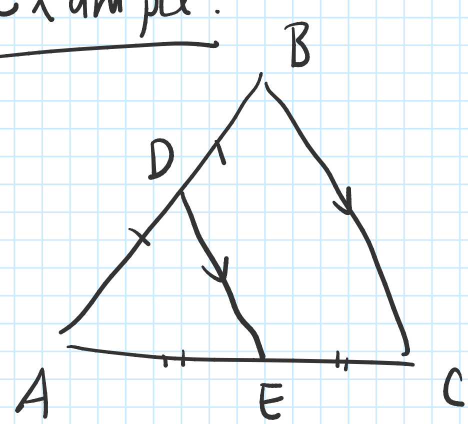


Then $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both have same length and direction and so are equal as vectors. This is why we usually replace $\overrightarrow{AB}$ with a letter.

ie $\vec{v} = \overrightarrow{AB}$ and as square, $\vec{v} = \overrightarrow{CD}$ also.

We can do geometry with vectors.

<u>Example.</u>



Here ABC is a Δ, $BC \parallel DE$ and D bisects AB, E bisects AC. We will show that $\overrightarrow{BC} = 2\overrightarrow{DE}$. In particular, $\|\overrightarrow{BC}\| = 2\|\overrightarrow{DE}\|$.

<u>proof:</u>

proof:

We have $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$ by subtraction

$= 2\overrightarrow{AE} - 2\overrightarrow{AD}$ as midpoints

$= 2(\overrightarrow{AE} - \overrightarrow{AD})$

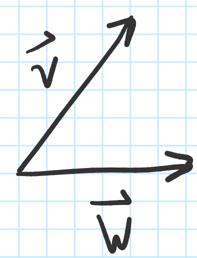
$= 2\overrightarrow{DE}$ by subtraction.

∎

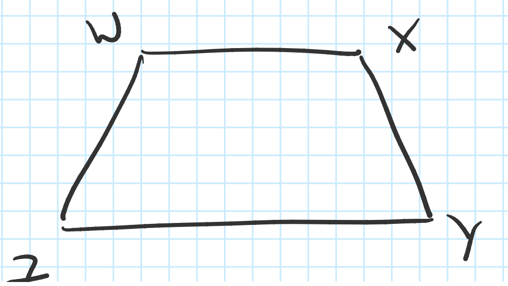
Since this is the first week, I want to do some questions as groups.

Questions:

1) Given $\vec{v}, \vec{w}$ as follows. Sketch: $-\vec{v}$, $2\vec{w}+\vec{v}$, $\vec{v}-\vec{w}$.

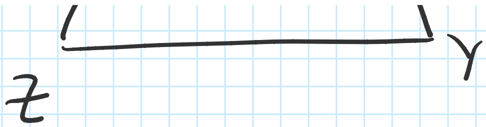


2) Given trapezium WXYZ:



where $ZY : WX = 3:2$.

let $s = \overrightarrow{WX}$, $t = \overrightarrow{WZ}$.

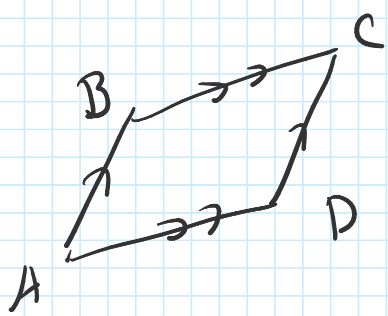


a) what is $\overrightarrow{ZY} = ?$ In terms of s, t

b) what is $\overrightarrow{XY} = ?$ in terms of s, t.

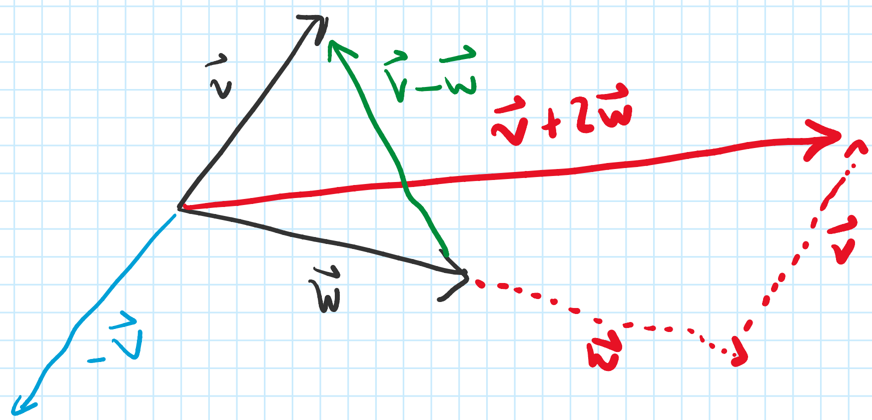
3) Is it always true that $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$?
why / why not?

4) Given a parallelogram ABCD. Prove that the diagonals bisect each other.



Answers:

1)



2) Since $ZY : WX = 3:2$ and $\vec{WX} = \vec{s}$ and $\vec{ZY}$ in same direction, we have $\vec{ZY} = \frac{3}{2} s$.

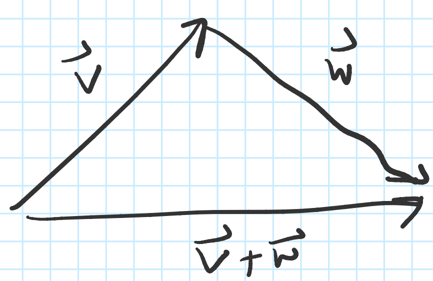
as for $\vec{XY}$, this is the same as going around the trapezium in the othe direction. so

$$\vec{XY} = \vec{XW} + \vec{WZ} + \vec{ZY}$$

$$= -s + t + \frac{3}{2} s$$

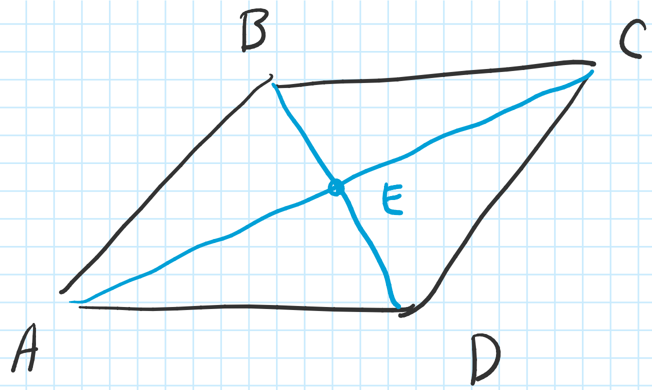
$$= t + \frac{1}{2} s.$$

3) Yes. This is called the triangle inequality.



$\vec{v}$, $\vec{w}$, $\vec{v} + \vec{w}$ form the edges of a triangle and the sum of any two side lengths is always greater than the third side length.

4) (This one was harder)



let the bisection point be E. The first step in these kind of questions is to reinterpret the geometry question into one about vectors.

The diagonals bisect each other if from A, we go half way to C lands us in the same place as if we started at B and went half way to D. ie

we want to show

$$\underbrace{\tfrac{1}{2}\overrightarrow{AC}}_{\substack{\text{half way}\\ \text{to C from}\\ \text{A.}}} = \underbrace{\overrightarrow{AB}}_{\substack{\text{get to}\\ \text{B from}\\ \text{A}}} + \underbrace{\tfrac{1}{2}\overrightarrow{BD}}_{\substack{\text{get half way}\\ \text{to D from B.}}} \quad \cdots \quad (1)$$

We will let $\vec{v} = \overrightarrow{AB}$, $\vec{w} = \overrightarrow{AD}$. Then $\overrightarrow{AC} = \vec{v} + \vec{w}$ by addition and $\overrightarrow{BD} = \vec{w} - \vec{v}$ by subtraction.

Now, in (1): $LHS = \frac{1}{2}(\vec{v} + \vec{w})$

$$\text{RHS} = \vec{v} + \frac{1}{2}(\vec{w} - \vec{v})$$
$$= \frac{1}{2}(\vec{w} + \vec{v})$$
$$= \text{LHS}.$$

Hence (1) is true and we are done. $\blacksquare$