Midterm 2 Practice

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Most of these problems come from Sang Truong's Midterm 2 problem set. I'll be posting answers to these at http://www.math.ucla.edu/~ben.szczesny/MATH32A-S19/coursehome.html

Question 1. The involute of a circle has parameterisation given by

$$\vec{r}(\theta) = \langle R(\cos(\theta) + \theta\sin(\theta), R(\sin(\theta) - \theta\cos(\theta))\rangle$$

Find the arclength parameterisation.

Question 2. Show that the curvature at an inflection point of a plane curve $y = f(x)$ is zero.

Question 3. Given a frenet frame $(\vec{T}, \vec{N}, \vec{B})$ with arclength paramareterisation.

(a) Show $\dfrac{d\vec{B}}{ds} = \vec{T} \times \dfrac{d\vec{N}}{ds}$ and conclude that $\dfrac{d\vec{B}}{ds}$ is orthogonal to $\vec{T}$.

(b) Show that $\dfrac{d\vec{B}}{ds}$ is orthogonal to $\vec{B}$. Hint: Differentiate $\vec{B} \cdot \vec{B} = 1$.

(c) Show that $\dfrac{d\vec{B}}{ds}$ is a multiple of $\vec{N}$.

Question 4. A particle has orbit given by

$$\vec{r}(t) = \langle \ln(t), t, t^2/2\rangle \text{ for } t \geq 0.$$

Find the equation for the osculating plane to this particle at $t = 1$

Question 5. Show that for a vector function $\vec{r}(t)$, both $\vec{r}\,'(t)$ and $\vec{r}\,''$ lie in the osculating plane. Hint: differentiate $\vec{r}\,'(t) = v(t)\vec{T}(t)$.

Question 6. Find the domain for the following functions

(a) $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2 - 1}}$

(b) $f(x, y) = \dfrac{y\sin(x)}{1 + y}$

(c) $f(x, y) = -\dfrac{1}{\sin(xy)}$