

Problem 1

$$g(x,y) = e^{xy} \text{ at } (2,1)$$

$$g_x(x,y) = \frac{1}{y} e^{xy} \text{ so } g_x(2,1) = e^2$$

$$g_y(x,y) = -\frac{x}{y^2} e^{xy} \text{ so } g_y(2,1) = -2e^2$$

Hence tangent plane is:

$$z = g(a,b) + g_x(a,b)(x-a) + g_y(a,b)(y-b)$$

$$z = e^2 + e^2(x-2) - 2e^2(y-1).$$

Problem 2

(a) Horizontal when $\frac{\partial f}{\partial x}(a,b) = \frac{\partial f}{\partial y}(a,b) = 0$

(b) rearranging the equation gives:

$$z - f_x(a,b)x - f_y(a,b)y = f(a,b) - f_x(a,b)a - f_y(a,b)b$$

Constant

Hence normal is: $\langle -f_x(a,b), -f_y(a,b), 1 \rangle$

(c) No. It is vertical if the normal is of the form $\langle a, b, 0 \rangle$ which can't happen.

Problem 3

$$f(1,2) = 4 \quad f_x = 3 \quad f_y = -2$$

(a) $f(2,2) = 4 + 1 \times 3 = 7$

since increase 1 in x-direction.

(b) $f(2,4) = 4 + 1 \times 3 + 2 \times (-2) = 3$

since increase 2 in x-direction and 2
in y-direction

(c) $f(4,1) - f(2,1) = 2 \times 3 = 6$

since increase 2 in x-direction.

Problem 4

$$f(x,y) = \sqrt{\frac{x}{y}}$$

$$f_x = \frac{1}{2\sqrt{\frac{x}{y}}} \cdot \frac{1}{y} \quad f_x(9,4) = \frac{1}{12}$$

$$f_y = \frac{1}{2\sqrt{\frac{x}{y}}} \cdot \left(-\frac{x}{y^2}\right) \quad f_y(9,4) = \frac{1}{3} \left(-\frac{9}{16}\right)$$
$$= -\frac{3}{16}$$

so linearization

$$L(x,y) = \frac{3}{2} + \frac{1}{12}(x-9) - \frac{3}{16}(y-4)$$

$$L(9.1, 3.9) = \frac{3}{2} + \frac{1}{12}(0.1) + \frac{3}{16}(0.1)$$

Problem S

(a) False. Essentially, the partials mean only differentiable from the x or y direction but differentiability is from all directions.

(b) True. This follows as $\lim f(x,y) - L(x,y) = 0$ and as $L(x,y)$ continuous at (a,b) , we have $\lim L(x,y) - L(a,b) = 0$ and as $L(a,b) = f(a,b)$ we have by limit laws:

$$\begin{aligned}\lim f(x,y) - f(a,b) &= \lim f(x,y) - L(x,y) + L(x,y) - L(a,b) \\&= \lim (f(x,y) - L(x,y)) \\&\quad + \lim (L(x,y) - L(a,b)) \\&= 0.\end{aligned}$$

Hence $\lim f(x,y) = f(a,b)$.