Problem 1 A bee with velocity vector $\mathbf{r}'(t)$ starts out at the origin at $t = 0$ and flies around for T seconds. Where is the bee located at T seconds if $\int_0^T \mathbf{r}'(u)du = 0$?. What does the quantity $\int_0^T ||\mathbf{r}'(u)||du$ represent?

Problem 2 In this question we will find the arclength parameterization of $\mathbf{r}(t) = \langle e^t \sin(t), e^t \cos(t), e^t \rangle$.

(a) Find $||\mathbf{r}'(t)||$.

(b) Find arclength s as a function of time t by using

$$s(t) = \int_0^t ||\mathbf{r}'(u)||du.$$

(c) Find time t as a function of arclength s by inverting the previous function.

(d) What is an arclength parameterization of $\mathbf{r}(t)$?

Problem 3 Determine whether the following are true or false.

(a) A circle of radius 3 has more curvature than a circle of radius 4.

(b) Reparameterizing a curve can change it's curvature at a point.

(c) A curve with constant curvature is a circle.

Problem 4 For this question, consider the ellipse given by the equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Sketch this curve. When do you think the curvature maximal? When is it minimal?

(b) Consider the plane as the xy-plane in 3-space. The ellipse then has parameterisation given by

$$\mathbf{r}(t) = \langle 3\cos(t), 2\sin(t), 0 \rangle.$$

Use the formula

$$\kappa(t) = \frac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$$

to calculate the curvature.

(c) Does your answer in the previous part agree with your guess form the first?