

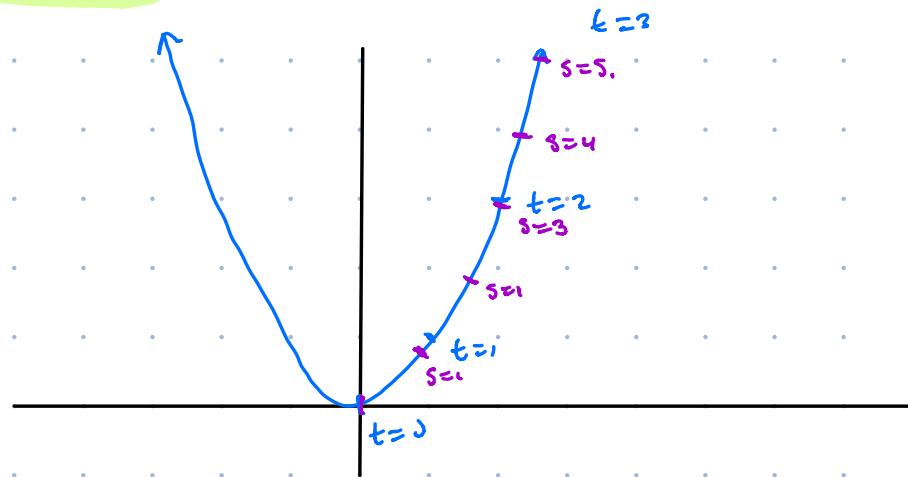
## Arclength Parameterization

Normally we think of a parameterization as with respect to time. Instead we can do so with respect to arclength.

That is,  $\vec{r}(s)$  gives the position of the point after moving  $s$  units along the curve determined by  $\vec{r}(s)$ .

Alternatively, the parameterisation given by moving along the curve at a constant speed of 1.

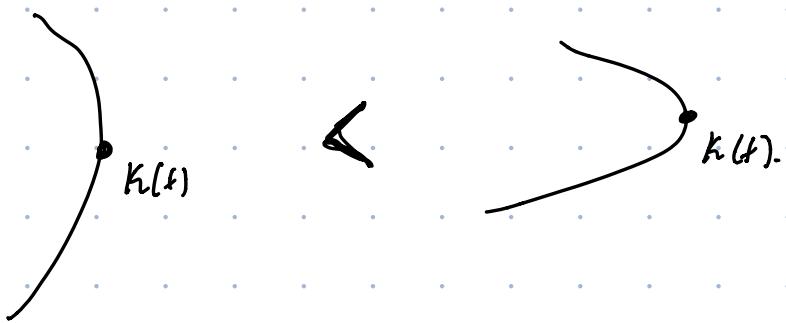
**Example:** Consider  $\vec{r}(t) = \langle t, t^2 \rangle$ .



$\vec{r}_1(s)$  is arclength parameterization. Notice how the distance travelled at each interval travelled is the same.

## Curvature

Curvature is a measurement of "how much a curve is bending" at a point. If it's positive and larger values mean its more curved.



**Definition:** Given arclength parameterization  $r(s)$ , then Curvature is

$$k(s) = \left\| \frac{dT}{ds} \right\|$$

where  $T$  is unit tangent.

- $T$  always has unit length, so it's rate of change comes from how fast its moving direction.

- Difficult to use this for actual calculation.

Instead,

$$K(f) = \frac{\| \vec{F}'(f) \times \vec{r}''(f) \|}{\| \vec{r}'(f) \|^3} \quad \text{for any param. } \vec{r}(f).$$

Alternate view of curvature: At each point there is a tangent circle called the osculating circle. The curvature is  $1/R$  where  $R$  is the radius of this circle.

