

## Problem 1

We find when the particle is at  $(0, 1, \pi/2)$

$$\vec{r}(t) = \langle 2\cos(t), \sin(t), t \rangle = (0, 1, \pi/2)$$

$$\therefore t = \pi/2$$

Now, the velocity vector is given by

$$\vec{v}(t) = \vec{r}'(t) = \langle -2\sin(t), \cos(t), 1 \rangle$$

and at  $t = \pi/2$ :

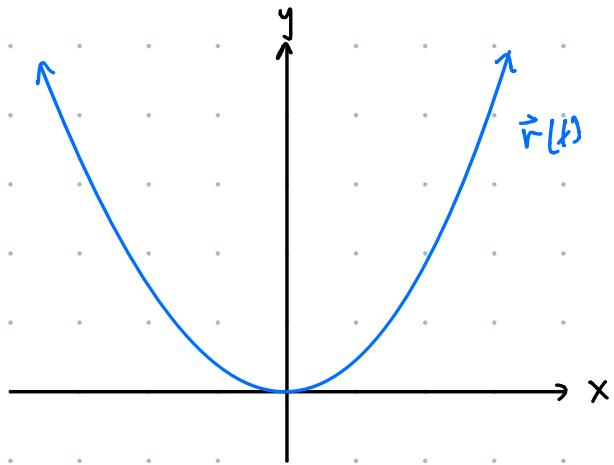
$$\vec{v}(\pi/2) = \langle -2, 0, 1 \rangle$$

Hence we get the tangent line as:

$$\vec{l}(t) = \langle 0, 1, \pi/2 \rangle + t \langle -2, 0, 1 \rangle$$

## Problem 2

(a)  $\vec{r}(t) = \langle t^3, t^6 \rangle$ . This is  $y = t^6$ ,  $x = t^3 \Rightarrow y = x^2$  so,



(b)  $\vec{v}(t) = \vec{r}'(t) = \langle 3t^2, 6t^5 \rangle$  so  $\vec{v}(0) = \vec{0}$

(c) The curve should have a tangent at  $t=0$ , as  $\vec{r}(0) = \langle 0, 0 \rangle$  is the origin, and by the picture has tangent line which is given by the  $x$ -axis  $y=0$ .

(d) Take  $s=t^3$ . Then  $\vec{r}(s) = \langle s, s^2 \rangle$  and this is such  $\vec{r}'(s) = \langle 1, 2s \rangle$  and  $\vec{r}'(0) = \langle 1, 0 \rangle$ . Hence this gives tangent line:

$$\vec{l}(t) = \langle 0, 0 \rangle + t \langle 1, 0 \rangle = \langle t, 0 \rangle$$

a) expected.

### Problem 3

(a) If  $\|\vec{r}(t)\| = 0$  then  $\|\vec{r}(t)\|$  is constant and so the magnitude of position stays constant. Hence the particle stays on the circle of radius  $\|\vec{r}(t)\|$ .

(b) If  $\|\vec{r}'(t)\| = 0$  then  $\vec{r}'(t) = \vec{0}$  since only the zero vector has zero magnitude. As  $\vec{r}'(t) = \vec{0}$  this implies that  $\vec{r}(t)$  is constant since its velocity is zero, i.e., it doesn't move.

### Problem 4

The condition  $\vec{r}'(t) \cdot \langle 1, 0, 0 \rangle > 0$  means that for all  $t$ ,  $\vec{r}'(t)$  makes an acute angle with the vector  $\langle 1, 0, 0 \rangle$ . So the velocity of the point is always slanted at least a little towards the positive x-axis.

Since it starts at the origin, to collide with  $(-2, 0, 3)$  would require it to move in the general direction of the negative x-axis which is impossible.

## Problem 5

Reparameterisations can only change the magnitude of velocity vectors but not the direction. So  $\vec{r}_1'(0)$ , and  $\vec{r}_2'(0)$  are parallel and so  $\vec{r}_1'(0) \times \vec{r}_2'(0) = 0$ .