

1. a) Let θ be the angle between \vec{u}, \vec{v} .
 $\vec{u} \cdot \vec{v}$ is negative when θ is obtuse
positive — " — acute
zero — " — orthogonal

$$\begin{aligned} b) (\vec{v} + 2\vec{w}) \cdot \vec{u} - \vec{u} \cdot \vec{v} &= \vec{v} \cdot \vec{u} + 2\vec{w} \cdot \vec{u} - \vec{u} \cdot \vec{v} \\ &= \vec{u} \cdot \vec{v} + 2\vec{w} \cdot \vec{u} - \vec{u} \cdot \vec{v} \\ &= 2\vec{w} \cdot \vec{u}. \end{aligned}$$

c) $\vec{u} \cdot \vec{v}$ is a scalar while \vec{v} is a vector.

$$\begin{aligned} 2. \quad \|\vec{e} + \vec{f}\|^2 &= (\vec{e} + \vec{f}) \cdot (\vec{e} + \vec{f}) \\ &= \vec{e} \cdot \vec{e} + \vec{f} \cdot \vec{e} + \vec{e} \cdot \vec{f} + \vec{f} \cdot \vec{f} \\ &= \|\vec{e}\|^2 + 2\vec{e} \cdot \vec{f} + \|\vec{f}\|^2 \\ \text{since } \|\vec{e}\| = \|\vec{f}\| = 1 \text{ and } \|\vec{e} + \vec{f}\| = \frac{\sqrt{5}}{2} \end{aligned}$$

$$\begin{aligned} \therefore \frac{9}{4} &= 1 + 2\vec{e} \cdot \vec{f} + 1 \\ 2\vec{e} \cdot \vec{f} &= \frac{1}{4} \\ \vec{e} \cdot \vec{f} &= \frac{1}{8}. \end{aligned}$$

$$\begin{aligned} \|\vec{e} - \vec{f}\|^2 &= \|\vec{e}\|^2 - 2\vec{e} \cdot \vec{f} + \|\vec{f}\|^2 \\ &= 1 - \frac{1}{4} + 1 \\ &= \frac{7}{4} \end{aligned}$$

Hence $\|\mathbf{e} - \mathbf{f}\| = \frac{\sqrt{7}}{2}$

3. (a) $(\vec{u} \cdot \vec{v}) \times \vec{w}$ doesn't make sense as $\vec{u} \cdot \vec{v}$ is a scalar and you can't take the cross product with a scalar

$$(\vec{u} \times \vec{v}) + \vec{v} \times \vec{u} = \vec{u} \times \vec{v} - \vec{u} \times \vec{v} = 0$$

(Swapping order in cross product changes the sign)

(c) No. Consider $\vec{u} = i, \vec{v} = i, \vec{w} = j$

$$(i \times i) \times j = 0 \times j = 0$$

$$i \times (i \times j) = i \times k = -i$$

4. We first find a vector orthogonal by taking cross product

$$\langle 3, 1, 1 \rangle \times \langle -1, 2, 1 \rangle = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} = \langle -1, -4, 7 \rangle.$$

Now, take \pm of the unit vector

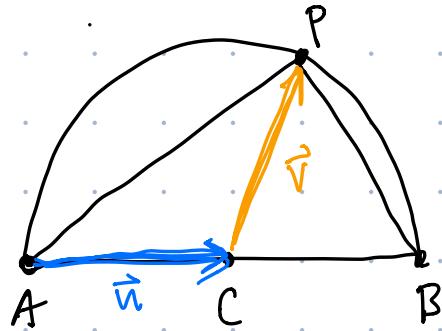
$$\|\langle -1, -4, 7 \rangle\| = \sqrt{1+16+49} = \sqrt{66}.$$

Hence $\pm \frac{1}{\sqrt{66}} \langle -1, -4, 7 \rangle$ are the vectors we want.

5. The volume of the spanning parallelepiped of 3 vectors $\vec{u}, \vec{v}, \vec{w}$ is given by the "scalar triple product" $\vec{u} \times (\vec{v} \cdot \vec{w})$

Geometrically, \vec{u}, \vec{v} and \vec{w} lie on the same plane iff the spanning parallelopiped is "flat" ie has zero volume.
This is true if and only if $\vec{u} \times (\vec{v} \cdot \vec{w}) = 0$.

6



Let C be the center of the circle, P the point on the circumference, A, B the ends of the diameter.

Let $\vec{u} = \vec{AC}$ and $\vec{v} = \vec{CP}$. We want to show that $\vec{AP} \cdot \vec{PB} = 0$
Now, notice that $\vec{AP} = \vec{u} + \vec{v}$ and $\vec{PB} = \vec{CB} - \vec{CP} = \vec{u} - \vec{v}$. Note, $\vec{CB} = \vec{u}$ since \vec{CB} has same direction and length as \vec{AC} .

$$\begin{aligned} \text{We then have } \vec{AP} \cdot \vec{PB} &= (\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) \\ &= \vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{u} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{v} \\ &= \|\vec{u}\|^2 - \|\vec{v}\|^2 \\ &= 0 \end{aligned}$$

Since \vec{u}, \vec{v} both have same length as the radius. ■