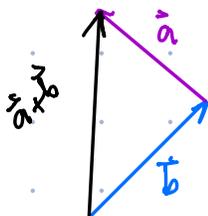
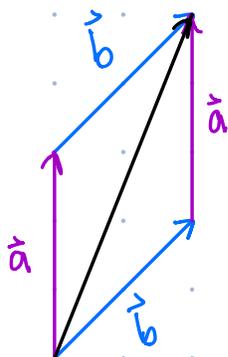


## Problem 1

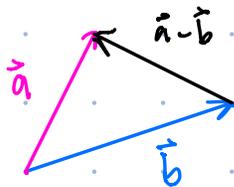
(a) Geometrically,  $\vec{a} + \vec{b}$  is the vector given by attaching the tail of  $\vec{a}$  to the head of  $\vec{b}$



If we compare  $\vec{a} + \vec{b}$  to  $\vec{b} + \vec{a}$ , these form the edges of a parallelogram and both  $\vec{a} + \vec{b}$ ,  $\vec{b} + \vec{a}$  is the long diagonal, and so equal.



(b) No. Geometrically  $\vec{a} - \vec{b}$  is the vector with tail at the head of  $\vec{b}$  and head at the head of  $\vec{a}$



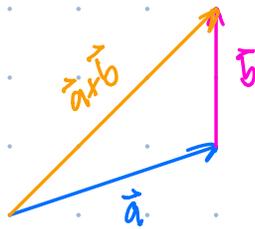
So we see that  $\vec{b} - \vec{a}$  is actually the vector  $\vec{a} - \vec{b}$  but in the opposite direction. i.e.  $\vec{b} - \vec{a} = -(\vec{a} - \vec{b})$ .

(c)  $\| -5\vec{a} \| = |-5| \| \vec{a} \| = 5 \cdot 5 = 25$

(d) This is given by the vector  $\vec{b} - \vec{a} = \langle -1, 3 \rangle - \langle 2, 1 \rangle = \langle -3, 2 \rangle$

## Problem 2

(a) Consider the picture:



It is not hard to see that the only way the length of  $\vec{a} + \vec{b}$  is equal to the length of  $\vec{a}$  and  $\vec{b}$  is if  $\vec{a}$  and  $\vec{b}$  point in the same direction. i.e.,  $\| \vec{a} + \vec{b} \| = \| \vec{a} \| + \| \vec{b} \|$  iff  $\vec{a}$  and  $\vec{b}$  point in the same direction.

(b) The only time that the square of one side of a  $\Delta$  is equal to the other two sides squared is when it's a right angle  $\Delta$  (Pythagoras, or law of cosines). Hence  $\| \vec{a} + \vec{b} \|^2 = \| \vec{a} \|^2 + \| \vec{b} \|^2$  iff  $\vec{a}$  and  $\vec{b}$  are orthogonal.

(c) As  $\|\vec{u}\| + \|\vec{v}\| = \|\vec{u} + \vec{v}\|$ ,  $\vec{u}$ ,  $\vec{v}$  point in the same direction. Hence  $\vec{v} = \lambda \vec{u}$  for some positive number  $\lambda$ .

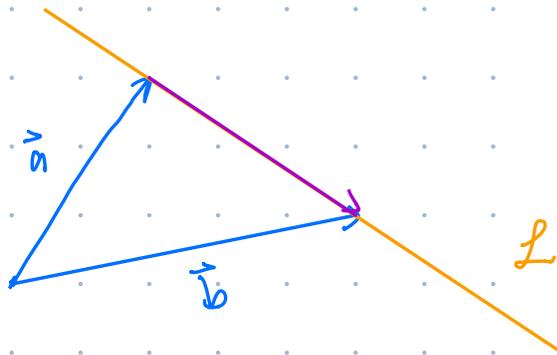
As  $\|\vec{u} + \vec{v}\| = \|\vec{u} + \lambda \vec{u}\| = |1+\lambda| \|\vec{u}\|$  and  $\|\vec{u} + \vec{v}\| = 15$  is given, we have  
 $|1+\lambda| \|\vec{u}\| = 15$

Since  $\|\vec{u}\| = \sqrt{4^2 + 3^2 + 0^2} = 5$ , we have  $(1+\lambda)5 = 15 \Rightarrow 1+\lambda = 3$   
and so  $\lambda = 2$ .

Hence  $\vec{v} = 2\vec{u} = \langle 8, 6, 0 \rangle$ .

### Problem 3

(a) Consider:



So the vector  $\vec{b} - \vec{a}$  should be in the same direction as the line.  
ie let  $\vec{u} = \vec{b} - \vec{a} = \langle 2, 1, -1 \rangle - \langle 1, 0, 1 \rangle = \langle 1, 1, -2 \rangle$

(b) For a  $t$ ,  $\vec{c} + t\vec{u}$  is given by attaching the vector  $t\vec{u}$  at the head of  $\vec{c}$ . As  $\vec{u}$  is in the direction of the line,  $\vec{c} + t\vec{u}$  must still be on the line. As  $t$  varies, all these must still then also be on

the line, and in fact must go over all pts on the line.

For instance, take  $\vec{c} = \vec{a} = \langle 1, 0, 1 \rangle$ . Then an equation for the line is:

$$f(t) = \vec{a} + t\vec{u} = \langle 1, 0, 1 \rangle + t\langle 1, 1, -2 \rangle = \langle 1+t, t, 1-2t \rangle.$$

(c). Yes. Any choice of vector  $\vec{u}$  in the direction of the line and point  $\vec{c}$  on the line gives a parametric equation for the line.

### Problem 4

(a) From Problem 2, we know  $\|\vec{u}\|^2 + \|\vec{v}\|^2 = \|\vec{u} + \vec{v}\|^2$  if  $\vec{u}, \vec{v}$  are orthogonal.

$$\begin{aligned} \text{Now, } \|\vec{u} + \vec{v}\|^2 &= \|\langle a+c, b+d \rangle\|^2 \\ &= (a+c)^2 + (b+d)^2 \\ &= a^2 + 2ac + c^2 + b^2 + 2bd + d^2 \\ &= \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2(ac+bd) \\ &= \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2D \end{aligned}$$

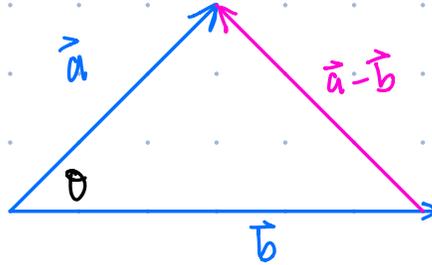
Hence,  $\vec{u}, \vec{v}$  orthogonal  $\Leftrightarrow D=0$ .

(b) From (a) we have that  $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2D$ . Since in the same direction,  $\|\vec{u} + \vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$  and so we have that:

$$\begin{aligned} (\|\vec{u}\| + \|\vec{v}\|)^2 &= \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2D \\ \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\|\vec{u}\|\|\vec{v}\| &= \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2D \end{aligned}$$

Hence  $D = \|\vec{u}\|\|\vec{v}\|$ .

(c) In general we have that



The law of cosines gives us

$$\|\vec{a} - \vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\|\vec{a}\|\|\vec{b}\|\cos\theta$$

$$\begin{aligned} \text{and as } \|\vec{a} - \vec{b}\|^2 &= \|\langle a-c, b-d \rangle\|^2 \\ &= (a-c)^2 + (b-d)^2 \\ &= a^2 - 2ac + c^2 + b^2 - 2bd + d^2 \\ \therefore \|\vec{a} - \vec{b}\|^2 &= \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2D \end{aligned}$$

Hence comparing these two expressions for  $\|\vec{a} - \vec{b}\|^2$  gives

$$D = \|\vec{a}\|\|\vec{b}\|\cos\theta.$$