Problem 1 For these questions, consider vectors $\mathbf{a}$ and $\mathbf{b}$ in the plane $\mathbb{R}^2$.

(a) Use the parallelogram law to explain geometrically why $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$. That is, the order in which you add vectors doesn't matter.

(b) Is it true that $\mathbf{a}-\mathbf{b}=\mathbf{b}-\mathbf{a}$? Why, why not?

(c) Suppose that $||\mathbf{a}||=5$. What is the length of $-5\mathbf{a}$?

(d) Suppose that $\mathbf{a}=\langle 2,1\rangle$ and $\mathbf{b}=\langle -1,3\rangle$ anf both are based at the origin. Compute the vector that connects the head of $\mathbf{a}$ to the head of $\mathbf{b}$.

Problem 2 For these questions, consider vectors $\mathbf{u}$ and $\mathbf{v}$ in the plane $\mathbb{R}^3$.

(a) When is it true that $||\mathbf{u}||+||\mathbf{v}||=||\mathbf{u}+\mathbf{v}||$?

(b) What about $||\mathbf{u}||^2+||\mathbf{v}||^2=||\mathbf{u}+\mathbf{v}||^2$?

(c) Suppose we have that $||\mathbf{u}||+||\mathbf{v}||=||\mathbf{u}+\mathbf{v}||$, $||\mathbf{u}+\mathbf{v}||=15$ and $\mathbf{u}=\langle 4,3,0\rangle$. What is $\mathbf{v}$?

Problem 3 [1] Here we find the parametric equations for a line in $\mathbb{R}^3$ passing through the points $\mathbf{a}=\langle 1,0,1\rangle$ and $\mathbf{b}=\langle 2,1,-1\rangle$.

(a) Find a vector $\mathbf{u}$ in the same direction as the line.

(b) Let $\mathbf{c}$ be any point on the line. Explain why $\mathbf{c}+t\mathbf{u}$ gives a parametric equation for the line. Write down this equation.

(c) Can you get more than one parametric equation for the same line through these methods?

Problem 4 (Additional, harder problem) Consider two vectors $\mathbf{u}=\langle a,b\rangle$ and $\mathbf{v}=\langle c,d\rangle$. We will consider the number $D=ac+bd$. *Hint: Consider Question 2*

(a) Show that $D=0$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

(b) Show that $D=||\mathbf{u}||||\mathbf{v}||$ if $\mathbf{u}$ and $\mathbf{v}$ point in the same direction.

(c) Use the law of cosines to find a formula for D in terms of the lengths of $\mathbf{u}$, $\mathbf{v}$ and the angle θ between them.

[1] From https://math.berkeley.edu/ hutching/teach/53-2015/53worksheets.pdf