Problem 1 Get to know each other in your groups! Here are some things to talk about:

- (a) Name and majors.
- (b) What other classes are you taking this quarter?
- (c) How do you plan to deal with remote learning and keeping motivated?
- (d) Do you have any study strategies for maths?
- (e) Do you know any good resources you can share with each other?
- Problem 2 Can you explain the following things about the derivative? If you don't remember, ask your group!
 - (a) How is the derivative of a function defined?
 - (b) There are two main interpretations of what the derivative represents: the slope of the tangent line and as rate of change. Can you explain what these are in more detail?
 - (c) Can you come up with an example of a function that has a point in which no derivative exists?
- Problem 3 A real function in two variables is one that takes two real numbers as inputs and outputs a real number. These are written like:

$$f(x,y) = x^2 + 3y^2.$$

This means the function in which we input say x = 3, y = -1, then the output is $3^2 + 3(-1)^2 = 12$. This is written as

$$f(3,-1) = 3^2 + 3(-1)^2 = 12.$$

- (a) For the above function f(x, y), what are f(1, 2) and f(-1, -1)?
- (b) We can keep one of the input variables constant, and leave the other free. This gives us functions in one variable. What are the functions f(2, y) and f(x, 0)?
- **Problem 4** You have encountered functions in more than one variable before, they just weren't called that. For instance, the volume of a circular cone is given by the formula:

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius of the base circle and h is the height of the cone. The formula says that the volume V of a cone depends on *two variables*; r and h. We can rewrite this in our function notation:

$$V(r,h) = \frac{1}{3}\pi r^2 h.$$

(a) If I keep the height constant at h = 2, what is the rate of change of the volume with respect to the radius when r = 1? i.e., $\frac{dV}{dr}\Big|_{r=1}$.

- (b) Similarly, if I keep the radius constant at r = 1, what is the rate of change of the volume with respect to the height at h = 2? i.e, $\frac{dV}{dh}\Big|_{h=2}$.
- (c) What if instead of keeping one variable constant, we make one depend on another. For instance, suppose we impose the restriction that the height must always be twice the size of the radius. i.e, h = 2r. What then is $\frac{dV}{dr}\Big|_{r=1}^{2}$?
- (d) Discuss: You should have that your answer from Problem (4.a) is different to Problem (4.c). Both are measuring the rate of change of volume V with respect to radius r when r = 1 and h = 2 and yet they are different. Why do you think this is the case?

In some sense the derivative $\frac{dV}{dr}$ isn't quite good enough when dealing with multivariable functions as it leads to the problems we just got. During this course, we will be building better notation and mathematical constructions to deal with these issues.

Problem 5 The first part of the course deals with vectors, and so some knowledge and review of geometry is helpful.

(a) Suppose we have a parallelogram *ABCD*. Prove that the area is equal to the length of two adjacent edges and sine of their angle multiplied together. That is

Area =
$$|AB||AD|\sin(\theta)$$
.

You may assume that a parallelogram area formula Area = bh.

(b) Suppose we have two points in \mathbb{R}^2 , $P = (x_0, y_0)$ and $Q = (x_1, y_1)$. The length between these two points is given by |PQ|. i.e,

$$|PQ| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}.$$

Show that

$$x_0 x_1 + y_0 y_1 = |OP||OQ|\cos(\theta)$$

where θ is the angle $\angle POQ$ and O is the origin O = (0,0). You may assume the law of cosines which says that for a triangles with side lengths A, B, C and opposite angles a, b, c to those sides, then

$$C^2 = A^2 + B^2 - 2AB\cos(c).$$