Math 31B: Week 9 Section

TA: Ben Szczesny Last updated: 2018/03/05

Discussion Questions

Question 1. We want to figure out whether $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$ converges or diverges.

(a) What is wrong with the following argument?

Since $(\ln(n))^2 \geq 1$, we have that $\frac{1}{n(\ln(n))^2} \leq \frac{1}{n}$. Hence by the direct comparison test, the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$ diverges.

(b) What is the integral test?

(c) Use the integral test to determines whether $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$ diverges or converges.

Question 2. We investigate the convergence of the series $\sum_{n=1}^{\infty}(-1)^n a_n$ where

$$a_n = \begin{cases} \frac{2}{n+1} & \text{when } n \text{ odd} \\ \frac{1}{2^{n/2}} & \text{when } n \text{ even.} \end{cases}$$

i.e,

$$\sum_{n=1}^{\infty}(-1)^n a_n = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{8} + \cdots$$

(a) Prove that this series is not absolutely convergent.

(b) What is wrong with the following argument?

Since this is an alternating series, by the alternating series test the series $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(c) Does this series converge or diverge? Hint: Try and sum the negative terms of the series, what are you left with?

Question 3. Show that $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges. You may use that $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$.

Extra Questions

Question 4. determine convergence or divergence of the following.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$