Math 31B: Week 8 Section

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Information

Discussion Questions

Question 1. The formal definition of limit is one of the hardest things to understand when first encountered.

One interesting way to think of limits is as a game[1]. The game is as follows:
Set up: We have two players A and B as well as a sequence (a_n) and a number L.

1. Player B picks a number $\epsilon > 0$ (preferably small).
2. Player A then picks an integer $M > 0$ (preferably large).
3. Player B then picks an integer N larger than M.

The value $|a_N - L|$ is then checked. If it is larger than ϵ, player B wins. If it is smaller than ϵ, player A wins. Then $\lim_{n\to\infty} a_n = L$ is the same thing as player A can always win, while $\lim_{n\to\infty} a_n \neq L$ means Player B can always win (assuming both players are playing smartly).

(a) Just to get a bit of practice with the game, find a partner and play against them with $a_n = \frac{n+4}{n+1}$ and $L = 1$. Who do you expect to win?

(b) With the same a_n and L as the previous question. Suppose player B picks $\epsilon = 1/5$. What M should player A pick to ensure that they win the game?

(c) Suppose we have the sequence $a_n = (-1)^n$ and $L = 1$. What value for ϵ should player B pick to ensure that he wins?

Question 2. Determine the limit of the following sequences as $n \to \infty$.

(a) $a_n = \sqrt{4 + \frac{1}{n}}$

(b) $a_n = \sqrt{n+3} - \sqrt{n}$

Question 3. Use partial fractions to rewrite $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ as a telescoping series and find it's value.

Homework Questions

Section 11.1
18, 26, 32, 40, 54, 62, 66, 70, 73, 81, 83
Section 11.2
14, 18. 22, 26, 34, 42, 46, 49, 53, 58, 59

[1] Adapted from https://cs.stanford.edu/people/slingamn/limits.pdf, have a look if you have time

Extra Questions

Question 4. Let a_n be the sequence defined recursively as follows:

$$a_0 = 0, \quad a_{n+1} = \sqrt{2 + a_n}.$$

(a) Show that if $a_n < 2$, then $a_{n+1} < 2$.

(b) Show that if $a_n < 2$, then $a_n \leq a_{n+1}$.

(c) The previous parts imply that the sequence (a_n) is increasing and bounded above since $a_0 < 2$. Hence the sequence has a limit L. Find L by taking the limit of both sides of the recursion equation.

Question 5. Let a and b be digits from 0 to 9. Find a fraction that has repeating decimal expansion given by $0.abababababab\ldots$.