Math 31B: Week 6 Section

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Information

Discussion Questions

Question 1.

(a) Find the Taylor polynomial T_3 of $\cos(x)$ centered at $x = \pi/2$.

(b) Prove that the n-th Maclaurin polynomial of $\cos(x)$ is given by

$$T_n(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!}.$$

(c) Use the error bound

$$|T_n(x) - \cos(x)| \leq \frac{K|x-a|^{n+1}}{(n+1)!}$$

to find an n such that

$$|T_n(0.1) - \cos(0.1)| \leq 10^{-7}.$$

Question 2.

(a) Show that $\displaystyle\int_1^\infty \frac{dx}{x^3+1}$ converges by comparing it with $\displaystyle\int_1^\infty \frac{dx}{x^3}$.

(b) Show that $\displaystyle\int_e^\infty \frac{dx}{x\ln(x)}$ diverges.

Question 3. *Gabriel's Horn.* Let $f(x) = \dfrac{1}{x}$. We will show that some shapes can have infinite surface area but only finite volume.

(a) Show that the surface of revolution around the x-axis over the interval $[1, \infty)$ is given by

$$2\pi \int_1^\infty \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\, dx.$$

(b) Use the comparison test to show that this integral diverges.

(c) Show that the volume of revolution over this same interval is finite.

Homework Questions

Section 9.4
4, 14, 18, 20, 31, 36, 44, 49, 52
Section 8.7
1, 4, 5, 6, 14, 16, 26, 32, 38, 46, 50, 54, 60, 62, 66, 76

Extra Questions

* **Question 4.** Notice that as a consequence of the error bound[1], we have that the remainder of a taylor expansion around a satisfies $\lim_{x\to a} \frac{R_n(x)}{(x-a)^n} = 0$. This can be used to solve limits in a similar way to L'Hopitals rule.

(a) Use L'hopitals to find the limit $\lim_{x\to 0} \frac{\sin(x) - x}{x^3}$.

(b) We know from the first question that $\sin(x) = x - \frac{x^3}{3!} + R_3(x)$. Use this to find the limit $\lim_{x\to 0} \frac{\sin(x) - x}{x^3}$.

(c) Similarly, find the limit $\lim_{x\to 0} \frac{\ln(1-x) - x}{x^2}$ with Taylor polynomials.

[1] Assuming $f^{(n+1)}(x)$ exists and is continuous.