Math 31B: Week 5 Section

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Information

Discussion Questions

Question 1.

(a) What is the point of numerical integration?

(b) Describe in your own words what the Midpoint (M_N), Trapezoidal (T_N) and Simpson's (S_N) Rules are.

(c) What are the formulas for these rules? (Can you do so without looking at your notes?)

Question 2. Consider the definite integral $\displaystyle\int_2^5 \frac{1}{x}dx$. In this question we will investigate how well the Trapezoidal Rule (T_N) approximates this integral. The error bound is given by the formula

$$\text{error}(T_N) \leq \frac{K_2(b-a)^3}{12N^2}.$$

(a) Do you expect the Trapezoidal Rule to over or underestimate the definite integral? If so, why?

(b) Let $f(x) = \frac{1}{x}$, the constant K_2 is any number such that $|f''(x)| \leq K_2$ for all x in the interval we are integrating over. However we usually take it to be the the absolute value of the maximum of the second derivative, $|\max_{x\in[a,b]} f''(x)|$. Find the maximum of f'' and set K_2 to be the absolute value of this value.

(c) In the formula $b-a$ is the length of the interval we are integrating over. In this case we have $b-1 = 5-2 = 3$. Use this and the previous part to find a value of N for which $\text{error}(T_N) < 10^{-6}$.

Question 3. Compute the arc length of $y = \ln\left(\dfrac{e^x+1}{e^x-1}\right)$ over the interval $[1, 3]$.

Question 4. Compute the surface are of revolution about the x-axis for $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$ over the interval $[1, e]$.

Homework Questions

Section 8.9
12, 16, 34, 36, 38, 40
Section 9.1
2, 9, 14, 18, 21, 23, 28, 40, 42

Extra Questions

Question 5. Evaluate the following integrals

(a) $\displaystyle\int \frac{dx}{x^2+2x+5}$

(b) $\displaystyle\int \sin^5(x)\cos^2(x)dx$

(c)* $\displaystyle\int \sin^4(x)\cos^2(x)dx$

(d) $\displaystyle\int \sqrt{1+\sqrt{x}}dx$

(e)* $\displaystyle\int \frac{1}{\operatorname{sech}(x)}dx.$

(f)** $\displaystyle\int_0^{\pi/2} \frac{\sin(x)}{\cos(x)+\sin(x)}dx$

Hint for (f): Remember the trig identities $\sin(\pi/2 - x) = \cos(x)$.

Question 6. Find the surface area of the torus obtained by rotating the circle $x^2 + (y-b)^2 = r^2$ about the x-axis.