Math 31B: Week 1 Section

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Information

The main course webpage is CCLE:
https://ccle.ucla.edu/course/view/18W-MATH31B-4
You should read the syllabus posted if you have not already. Some important highlights are:

- Math questions and administrative questions that apply to more than one person should be asked on the CCLE discussion board.
- Homework is due during Friday lectures. Late homework must be emailed to Alex Austin within 24hrs and this incurs a 50% penalty.

This week I will be holding office hours on Thursday at 3pm in MS 3957. Please go to
https://goo.gl/forms/gKfpmXcUsPlJFIav1
to vote on what office hours suit you, as well as a few other questions about things we could do in future sections.
On my web page, you can find electronic versions of any worksheet from sections as well as solutions.
http://www.math.ucla.edu/~ben.szczesny/
If you forget the link, you could probably also find it by googling something like "ben szczesny ucla". At the moment it's not linked by the main course webpage.

Discussion Questions

Question 1. Find the derivative of the following functions:

(a) $f(x) = e^{x^2+2x-3}$,

(b) $f(t) = \dfrac{1}{1-e^{-3t}}$,

(c) $f(\theta) = \sin(e^{\theta})$,

(d) $f(x) = \dfrac{e^x}{3x+1}$.

Solution to Question 1.

(a) $(2x+2)e^{x^2+2x-3}$,

(b) $\dfrac{-3e^{-3t}}{(1-e^{-3t})^2}$,

(c) $\cos(e^{\theta})e^{\theta}$,

(d) $\dfrac{(3x-2)e^x}{(3x+1)^2}$.

Question 2. Find the critical points of the function $f(x) = \dfrac{e^x}{x}$ for $x > 0$ and determine whether they are local minima or maxima (or neither).

Solution to Question 2.
We have that

$$f'(x) = e^x\left(\frac{x-1}{x}\right)$$

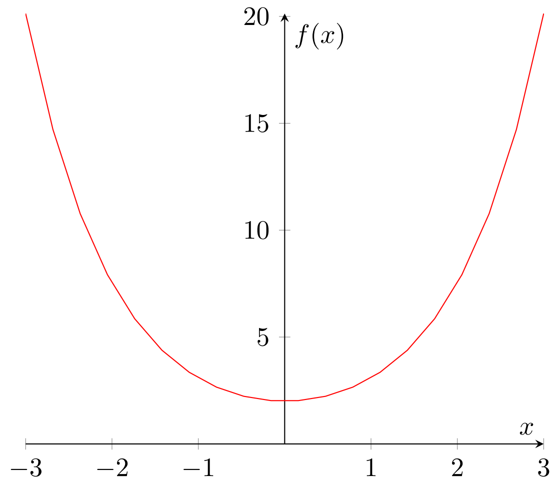
and so the only critical point is at $x = 1$. since $f' < 0$ for $x < 1$ and $f' > 0$ for $x > 1$, the first derivatives test tells us this is a local minimum. Alternatively, we can show that $f''(1) > 0$ which implies it is a local minimum by the second derivatives test.

Question 3. For $y = e^x + e^{-x}$, find critical points and points of inflection. Then sketch the graph.

Solution to Question 3.
We have that

$$\frac{dy}{dx} = e^x - e^{-x},$$
$$\frac{d^2y}{dx^2} = e^x + e^{-x}.$$

Hence, $f'(x) = 0$ if and only if $x = 0$ and so this is the only critical point. Since $f'' > 0$, this tells us there are no inflection points and that the critical point at $x = 0$ is a local minimum.



Question 4. Compute the linearisation of $f(x) = 2e^{-2x}\sin(x)$ at $a = 0$. Use a linear approximation to estimate $f(0.2) - f(0)$.

Solution to Question 4.
The linearisation $L(x)$ of a function $f(x)$ at a point is just the tangent line at that point,

$$L(x) = f(a) + f'(a)(x - a).$$

Hence, in this case we have that $f'(x) = -4e^{-2x}\sin(x) + 2e^{-2x}\cos(x)$ and so $f'(0) = 2$. Therefore,

$$L(x) = 2x.$$

The linear approximation is given by

$$\Delta f \approx f'(a)\Delta x$$

which in this case is

$$\begin{aligned} f(0.2) - f(0) &\approx f'(0)\Delta x \\ &= 2(0.2 - 0) \\ &= 0.4. \end{aligned}$$

Question 5. Evaluate the following integrals:

(a) $\displaystyle\int e^x + e^{-x} dx,$

(b) $\displaystyle\int e^x \cos(e^x) dx.$

Solution to Question 5.

(a) $e^x - e^{-x} + C$, (b) $\sin(e^x) + C$.

Homework Questions

Questions 14, 18, 26, 30, 34, 36, 40, 44, 50, 56, 62, 64, 72, 78, 88, 90, 92 of section 7.1 of the class textbook.

Extra Questions

Question 6. Find the Area bounded by $y = e^2, y = e^x$, and $x = 0$.

Solution to Question 6.
We have that $e^0 < e^2$ and so we see that the area is given by

$$\int_0^2 e^2 - e^x dx = e^2 + 1.$$

*** Question 7.** Prove that $f(x) = e^x$ is not a polynomial function. Hint: Differentiation lowers the degree of a polynomial by 1.

Solution to Question 7.
For any polynomial $P(x)$, we have that for a large enough n, $P^{(n)}(x) = 0$. However, $f^{(n)}(x) \neq 0$ for any $n \in \mathbb{N}$. Hence the exponential function can't possibly be a polynomial.

*** Question 8.** Define a function $A(x) := \int_1^x \frac{1}{t} dt$ for $x > 0$. Prove that $A(e^x) = x$. Hint: differentiate $A(e^x)$.

Solution to Question 8.
Differentiating with respect to x gives

$$\begin{aligned} \frac{d}{dx}\left(A(e^x)\right) =& A'(e^x)e^x \text{ by chain rule} \\ =& \frac{1}{e^x} e^x = 1 \text{ by FTC 2.} \end{aligned}$$

Hence we conclude $A(e^x) = x + C$ for some constant C. However, we know that $A(1) = 0$. Hence substituting $x = 0$, we see that $C = 0$.