

Math 31B: Mock Midterm 1

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Question 1. Differentiate the following with respect to x ,

- (a) $x^2 e^{2x}$
- (b) $e^{\sin x}$
- (c) $\tan(e^{5-6x})$
- (d) $e^{1/x}$
- (e) 4^{-2x}

Solution to Question 1.

- (a) $2x(x+1)e^{2x}$
- (b) $\cos(x)e^{\sin x}$
- (c) $-6 \sec^2(e^{5-6x})e^{5-6x}$
- (d) $\frac{-e^{1/x}}{x^2}$
- (e) $-2 \ln(4)4^{-2x}$

Question 2. Differentiate the following with respect to x ,

- (a) $x \ln(x) - x, x > 0$
- (b) $\ln((\ln x)^3), x > 0$
- (c) $\ln\left(\frac{x+1}{x^3+1}\right), x \neq -1$
- (d) $\frac{(x+12)^{5/2}}{(x-6)^{1/5}}, x \neq 6$

Solution to Question 2.

- (a) $\ln(x)$
- (b) $\frac{3}{x \ln(x)}$
- (c) $\frac{1}{x+1} - \frac{3x^2}{x^3+1}$
- (d) $\frac{5(x+12)^{3/2}}{2(x-6)^{1/5}} - \frac{(x+12)^{5/2}}{5(x-6)^{6/5}}$

Question 3.

1. For $1 < b$, is the function $f(x) = b^x$ increasing or decreasing?
2. Evaluate the limit $\lim_{x \rightarrow \infty} \ln \left(1 + \frac{1}{x} \right)$.
3. For $0 < b < 1$, is the function $f(x) = \log_b(x)$ for $x > 0$ increasing or decreasing?
4. Evaluate the indefinite integral $\int e^t \sqrt{e^t + 1} dt$.
5. Calculate $\int_e^{e^2} \frac{1}{t \ln(t)} dt$.

Solution to Question 3.

1. Increasing
2. 0
3. Decreasing
4. $\frac{2}{3}(e^t + 1)^{3/2} + C$
5. $\ln(2)$

Question 4.

1. Show that $f(x) = \frac{1}{1+x}$ and $g(x) = \frac{1-x}{x}$ are inverses.
2. Assume $f(x)$ is differentiable and one-to-one with inverse $g(x) = f^{-1}(x)$. Let b belong to the domain of g suppose $f'(g(b)) \neq 0$. Complete the formula:
$$g'(b) =$$
3. Suppose $f(x) = \arctan(e^x - e)$. Compute $f^{-1}(0)$ and $(f^{-1})'(0)$ with out explicitly finding the function $f^{-1}(x)$.
4. Use logarithmic differentiation to differentiate

$$f(x) = 2^x \cdot 3^{2x} \cdot e^{x^2}$$

Solution to Question 4.

1. Need to show that $g(f(x)) = x$ for $x \neq -1$ and $f(g(x)) = x$ for $x \neq 0$.
2. $g'(b) = \frac{1}{f'(g(b))}$.
3. $f^{-1}(0) = 1$, $(f^{-1})'(0) = e$.
4. $(2x + \ln(18))2^x \cdot 3^{2x} \cdot e^{x^2}$

Question 5. Evaluate the following limits:

1. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin(x)} - \frac{1}{x} \right)$
2. $\lim_{x \rightarrow 1} \left(\frac{x(\ln x - 1) + 1}{(x - 1) \ln x} \right)$
3. $\lim_{x \rightarrow 0} \left(\frac{\cos(x + \pi/2)}{\sin(x)} \right)$

Solution to Question 5.

1. 0
2. 1/2
3. -1

Question 6.

1. Differentiate $f(x) = e^{\arccos(x)}$.

2. Calculate the integral $\int_0^3 \frac{dx}{x^2 + 3}$

Recall: $\sinh(x) = (e^x - e^{-x})/2$, $\cosh(x) = (e^x + e^{-x})/2$, $\tanh(x) = \sinh(x)/\cosh(x)$ and we have the identity

$$\cosh^2(x) - \sinh^2(x) = 1.$$

3. What is $\lim_{x \rightarrow \infty} \tanh(\sinh(x))$?

4. Differentiate $\cosh^{-1}(x)$.

Solution to Question 6.

1. $\frac{-e^{\arccos(x)}}{\sqrt{1-x^2}}$
2. $\frac{1}{\sqrt{3}} \arctan(\sqrt{3})$
3. ∞
4. $\frac{1}{\sqrt{x^2-1}}$.

Question 7.

1. Evaluate $\int \arcsin^{-1}(x) dx$

2. Compute $\int_0^1 x e^{-x} dx$

Solution to Question 7.

1. $x \arcsin(x) + \sqrt{1-x^2} + C$

2. 1

Question 8.

1. Find the partial fraction decomposition of

$$\frac{3x^2 + 5x - 4}{(x-2)(x+1)^2}$$

2. Given that $\frac{2x^2 - 2x + 4}{(x-1)(x^2+1)} = \frac{2}{x-1} - \frac{2}{x^2+1}$, evaluate

$$\int \frac{2x^2 - 2x + 4}{(x-1)(x^2+1)} dx$$

Solution to Question 8.

1.

$$\frac{2}{x-2} + \frac{1}{x+1} + \frac{2}{(x+1)^2}$$

2. $2 \ln(x-1) - 2 \arctan(x) + C$