Math 31B: Final Practice Quiz

TA: Ben Szczesny Last updated: 2018/03/18

Time: 50 minutes. This tests sections 11.1-11.7 of textbook

Question 1.

(a) Let $a_n = (n + 10^n)^{1/n}$. Use the squeeze theorem to show that $\lim_{n\to\infty} a_n = 10$.

(b) What is the limit of $a_n = \ln(n^2 + 1) - \ln(n^2 - 1)$ as $n \to \infty$?

Question 2. For each of the following series, determine whether they diverge, converge conditionally or absolutely converge. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n^2$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \ln(n)}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\ln(n)}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

Question 3. Find the interval of convergence for the following power series

(a) $\displaystyle\sum_{n=15}^{\infty} \frac{x^{2n+1}}{3n+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} e^n (x-2)^n$

Question 4. Show, by integrating the Maclaurin series for $f(x) = \frac{1}{\sqrt{1-x^2}}$ that for $|x| < 1$,

$$\sin^{-1}(x) = x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \frac{x^{2n+1}}{2n+1}.$$