Week 1 Notes

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General Information:

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www.math.ucla.edu/~ben.szczesny/MATH31B-S18/coursehome.html.

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Crash course on Inverse functions. (for tuesday)

- A function $f: D \to R$ is a rule that takes something from domain D to range R
- A function is one-to-one (or injective) if you can't get the same output from two different inputs. ie $f(x) = x^2$ is

two different inputs. ie $f(x)=x^2$ is not one-to-one as $f(-2)=(-2)^2=4=f(2)$

- An inverse function is the function you get by doing the opposite rule ie, $f(x)=x^3$, the opposite is taking cube roots.

$$f^{-1}(x)=x^{1/3}$$ is the inverse function

- A function has an inverse iff it is one-to-one on its domain D. This is what the first qestion is about.

Example:

does the function $f(x)=x^{27}+4x$ have an inverse? One way to show a function is one-to-one is to show its strictly increasing/decreasing. We can do this by looking at its derivative (probably the most common f is differentiable (probably the most common way to do this)

$$f'(x) = 27x^{26} + 4 > 0 \text{ for all } x$$

Hence f is strictly increasing and so is one-to-one and has an inverse

Example

Find the inverse function of $f(x) = 3x - 9$.

We do so by solving $y = f(x)$ for y.

$$y = 3x - 9$$

$$y + 9 = 3x$$

$$\Rightarrow \frac{y+9}{3} = x$$

We then interchange x, y to get inverse

$$f^{-1}(x) = \frac{x+9}{3}.$$

Example:

For functions not one-to-one we can

restrict the domain so the function is one-to-one on the now smaller domain and then get an inverse for this new function. ie $f(x)=x^2$ not one-to-one on $\mathbb{R}$, but it is when domain is $[0,\infty)$ and it's inverse is then $f^{-1}(x)=\sqrt[2]{x}$

If we instead restricted the domain to $(-\infty,0]$, the inverse would be $f^{-1}(x)=-\sqrt[2]{x}$.

Log Rules

definition of $\log_b a$: the number that you raise b by to get a.

ie $b^{\log_b a} = a$

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^n) = n\log_b(x)$

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$\log_b 1 = 0 \qquad \log_b b = 1$

derivatives of exp/log

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}\ln(x) = \frac{1}{x}$$

when $\ln x = \log_e(x)$

- e is a super special number, in general

$$\frac{d}{dx}(b^x) \neq b^x, \quad \frac{d}{dx}\log_b(x) \neq \frac{1}{x}$$

- Remember the chain rule!

$$\frac{d}{dx}g(f(x)) = g'(f(x))f'(x).$$

Example

Derivative of $f(x) = e^{2x^2}$, $g(x) = \ln(2x)$

For first one we use chain rule:

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for $f(x) = e^{g(x)}$

$$f'(x) = e^{g(x)} \cdot g'(x)$$

so in our case

$$f'(x) = 4x e^{2x^2}$$

In second case, chain rule gives in general

$$g(x) = \ln(h(x))$$

$$g'(x) = \frac{h'(x)}{h(x)}$$

so in our case $g'(x) = \frac{2}{2x} = \frac{1}{x}$